

Influence of the back-reaction of streaming cosmic rays on magnetic field generation and thermal instability

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ABSTRACT

Using a multi-fluid approach, we investigate streaming and thermal instabilities of the electron-ion plasma with homogeneous cold cosmic rays propagating perpendicular to the background magnetic field. Perturbations are considered to be also across the magnetic field. The back-reaction of cosmic rays resulting in strong streaming instabilities is taken into account. It is shown that for sufficiently short wavelength perturbations, the growth rates can exceed the growth rate of cosmic-ray streaming instability along the magnetic field found by Nekrasov & Shadmehri (2012), which is in its turn considerably larger than the growth rate of the Bell instability (2004). The thermal instability is shown not to be subject to the action of cosmic rays in the model under consideration. The dispersion relation for the thermal instability has been derived which includes sound velocities of plasma and cosmic rays, Alfvén and cosmic-ray streaming velocities. The relation between these parameters determines the kind of thermal instability ranging from the Parker (1953) to the Field (1965) instabilities. The results obtained can be useful for a more detailed

investigation of electron-ion astrophysical objects such as supernova remnant shocks, galaxy clusters and others including the dynamics of streaming cosmic rays.

1. Introduction

Cosmic rays are an important ingredient in astrophysical environments (see, e.g., Zweibel 2003). They are capable of affecting the dynamics of astrophysical plasma media leading to plasma heating, increasing the level of ionization, driving outflows, modifying shocks, and so on (Zweibel 2003; Field et al. 1969; Guo & Oh 2008; Everett et al. 2008; Beresnyak et al. 2009; Samui et al. 2010; Enßlin et al. 2011). Cosmic-ray ionization contributes to star formation (e.g., Yusef-Zadeh et al. 2007) and coupling of gas to the magnetic field in accretion disks (Gammie 1996).

Thermal instability (Field 1965) have been used to explain existence of the cold dense structures in the interstellar (Field 1965; Begelman & McKee 1990; Koyama & Inutsuka 2000; Hennebelle & Pérault 2000; Sánchez-Salcedo et al. 2002; Vázquez-Semadeni et al. 2006; Fukue and Kamaya 2007; Inoue & Inutsuka 2008; Shadmehri et al. 2010) and intracluster (ICM; Field 1965; Mathews & Bregman 1978; Balbus & Soker 1989; Loewenstein 1990; Bogdanović et al. 2009; Parrish et al. 2009; Sharma et al. 2010) media. For example, molecular filaments have been observed in galaxy clusters by Conselice et al. (2001), Salomé et al. (2006), Cavagnolo et al. (2008), and O’Dea et al. (2008).

In galaxy clusters, cosmic rays are wide spread (e.g., Guo & Oh 2008; Enßlin et al. 2011). Therefore, they could exert influence on thermal instability. In particular, including cosmic rays is required to explain the atomic and molecular lines observed in filaments in clusters of galaxies by Ferland et al. (2009). Such an investigation has been performed by Sharma et al. (2010) in the framework of magnetohydrodynamic (MHD) equations.

Numerical analysis has shown that the cosmic-ray pressure can elongate cold filaments along the magnetic field lines. However in general, cosmic rays can be relativistic and have the streaming velocity of the order of the speed of light and the mean energy larger than the particle rest energy. The interaction of such particles with the thermal plasma can not be considered in the framework of the conventional MHD.

It is well-known that the cosmic-ray drift current results in arising of the return current in the background plasma and of streaming instabilities generating magnetic fields (Achterberg 1983; Zweibel 2003; Bell 2004, 2005; Riquelme & Spitkovsky 2009, 2010). In papers by Achterberg (1983), Zweibel (2003), and Bell (2004), the kinetic consideration of circularly-polarized electromagnetic waves traveling along the background magnetic field where cosmic rays also drift along the latter has been provided. For the case of the large cosmic-ray Larmor radius in comparison with the wavelength, Bell (2004) has found the growth rate some larger than that for the resonant cyclotron instability proposed long time ago by Kulsrud and Pearce (1969). The general case for the arbitrary mutual orientation of the background magnetic field, the cosmic-ray current and the wave vector of perturbations has been considered by Bell (2005) within the MHD framework. Riquelme & Spitkovsky (2010) have explored the case in which the cosmic-ray current is perpendicular to the initial magnetic field and perturbations are excited along the latter. In papers by Bell (2004, 2005) and Riquelme & Spitkovsky (2010), instabilities were excited due to the return plasma current and obtained growth rates were of the same order of magnitude. The dynamics of cosmic rays did not play the role (in the analytical consideration). Nekrasov & Shadmehri (2012) have included the back-reaction of cosmic rays in the multi-fluid approach for the model by Riquelme & Spitkovsky (2010) and found the growth rate for the streaming instability considerably larger than that of Bell (2004, 2005) and of Riquelme & Spitkovsky (2010) by a factor of the square root from the ratio of plasma to cosmic-ray number densities. The second result obtained by Nekrasov & Shadmehri (2012) was that

the thermal instability is not subject to the action of cosmic rays in the model considered. Instabilities along the background magnetic field driven by the back-reaction of relativistic cosmic rays drifting also parallel to the magnetic field have been considered by Nekrasov (2013).

These findings motivated us to investigate the case in which perturbations arise transversely to the ambient magnetic field in the directions both along and across the perpendicular cosmic-ray current. Such a current can appear due to diamagnetic drift of cosmic rays and inhomogeneity of the magnetic field (Bell 2005), due to gravitational cosmic-ray drift in magnetic field and so on. Riquelmi & Spitkovsky (2010) have discussed a possibility of an appearance of the perpendicular cosmic-ray current because of the magnetic wall effect of low-energy magnetized cosmic rays in the pre-amplified magnetic fields in the upstream medium of supernova remnant shocks. We note that such a mechanism can also operate in other cases in which cosmic rays encounter magnetic clouds. As it follows from Bell (2005), where the one-fluid MHD equations are used, the streaming instability does not exist for perturbations perpendicular to the magnetic field. However, this result is incorrect in the multi-fluid consideration (for three and more species) that is shown in this paper and has been obtained earlier (e.g., Nekrasov 2007). We here include the induced return current of the background plasma and back-reaction of cosmic rays. In this approach, dispersion relations are derived and growth rates are found analytically. We also consider possible effects of cosmic rays on the thermal instability for the geometry under consideration. We provide a comparison of results obtained in this paper with those of Nekrasov & Shadmehri (2012) and show a difference between them.

The paper is organized as follows. Section 2 contains the fundamental equations for plasma, cosmic rays, and electromagnetic fields used in this paper. The zero order state is discussed in Section 3. Wave equations are given in Section 4. In Sections 5 and 6,

the dispersion relations including the plasma return current, cosmic-ray back-reaction and the terms describing the thermal instability are derived and their solutions are found for perturbations along and across the cosmic-ray current, respectively. Discussion of important results obtained and possible astrophysical implications are provided in Section 7. Conclusive remarks are summarized in Section 8.

2. Basic equations for a plasma and cosmic rays

The fundamental equations for a plasma are the following:

$$\frac{\partial \mathbf{v}_j}{\partial t} + \mathbf{v}_j \cdot \nabla \mathbf{v}_j = -\frac{\nabla p_j}{m_j n_j} + \frac{q_j}{m_j} \mathbf{E} + \frac{q_j}{m_j c} \mathbf{v}_j \times \mathbf{B}, \quad (1)$$

the equation of motion,

$$\frac{\partial n_j}{\partial t} + \nabla \cdot n_j \mathbf{v}_j = 0, \quad (2)$$

the continuity equation,

$$\frac{\partial T_i}{\partial t} + \mathbf{v}_i \cdot \nabla T_i + (\gamma - 1) T_i \nabla \cdot \mathbf{v}_i = -(\gamma - 1) \frac{1}{n_i} \mathcal{L}_i(n_i, T_i) + \nu_{ie}^\varepsilon(n_e, T_e) (T_e - T_i) \quad (3)$$

and

$$\frac{\partial T_e}{\partial t} + \mathbf{v}_e \cdot \nabla T_e + (\gamma - 1) T_e \nabla \cdot \mathbf{v}_e = -(\gamma - 1) \frac{1}{n_e} \mathcal{L}_e(n_e, T_e) - \nu_{ei}^\varepsilon(n_i, T_e) (T_e - T_i) \quad (4)$$

are the temperature equations for ions and electrons. In Equations (1) and (2), the index $j = i, e$ denotes the ions and electrons, respectively. Notations in Equations (1)-(4) are the following: q_j and m_j are the charge and mass of species j ; \mathbf{v}_j is the hydrodynamic velocity; n_j is the number density; $p_j = n_j T_j$ is the thermal pressure; T_j is the temperature; $\nu_{ie}^\varepsilon(n_e, T_e)$ ($\nu_{ei}^\varepsilon(n_i, T_e)$) is the frequency of the thermal energy exchange between ions (electrons) and electrons (ions) being $\nu_{ie}^\varepsilon(n_e, T_e) = 2\nu_{ie}$, where ν_{ie} is the collision frequency of ions with electrons (Braginskii 1965); $n_i \nu_{ie}^\varepsilon(n_e, T_e) = n_e \nu_{ei}^\varepsilon(n_i, T_e)$; γ is the ratio of the

specific heats; \mathbf{E} and \mathbf{B} are the electric and magnetic fields; and c is the speed of light in vacuum. We include the thermal energy exchange because the corresponding frequency ν_{ie}^ε (ν_{ei}^ε) must be compared with the dynamical frequency for thermal instability. The cooling and heating of plasma species in Equations (3) and (4) are described by the function $\mathcal{L}_j(n_j, T_j) = n_j^2 \Lambda_j(T_j) - n_j \Gamma_j$, where Λ_j and Γ_j are the cooling and heating functions, respectively. This function has some deviation from the usually used cooling-heating function \mathcal{L} (Field 1965). Both functions are connected to each other via the equality $\mathcal{L}_j(n_j, T_j) = m_j n_j \mathcal{L}_j$. Our choice is analogous to those of Begelman & Zweibel (1994), Bogdanović et al. (2009), and Parrish et al. (2009). The function $\Lambda_j(T_j)$ can be found, for example, in Tozzi & Norman (2001). We do not take into account the transverse thermal fluxes in the temperature equations, which are small in a weakly collisional plasma (Braginskii 1965) being considered in this paper. For simplicity, we do not take into account a collisional coupling of ions and electrons in Equation (1). The corresponding condition will be given in Section 7.

The cosmic rays that we are interested in here, are considered as a possible source of the magnetic field generation and amplification in different astrophysical environments in which cosmic-ray fluxes may exist (Zweibel & Everett 2010) as well as their possible influence on thermal instability. It is important that cosmic rays have a drift velocity or a current relative to the direction of the background magnetic field and can excite instabilities due to their streaming. In this case, we are not interested in the cosmic-ray history, i.e., in the spatial and momentum diffusion of the quasi-isotropic cosmic-ray distribution function, described by the transport equation in the turbulent medium (e.g., Skilling 1975), and consider cosmic rays as beams governed by MHD equations in the vicinity of their local sources. Such an approach is adopted in the beam-plasma systems to study streaming instabilities. Equations for relativistic cosmic rays which can be in general both protons and electrons, we apply in the form of relativistic MHD equations given by Lontano et al.

(2001)

$$\frac{\partial (R_{cr} \mathbf{p}_{cr})}{\partial t} + \mathbf{v}_{cr} \cdot \nabla (R_{cr} \mathbf{p}_{cr}) = -\frac{\nabla p_{cr}}{n_{cr}} + q_{cr} \left(\mathbf{E} + \frac{1}{c} \mathbf{v}_{cr} \times \mathbf{B} \right), \quad (5)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_{cr} \cdot \nabla \right) \left(\frac{p_{cr} \gamma_{cr}^{\Gamma_{cr}}}{n_{cr}^{\Gamma_{cr}}} \right) = 0, \quad (6)$$

where

$$R_{cr} = 1 + \frac{\Gamma_{cr}}{\Gamma_{cr} - 1} \frac{T_{cr}}{m_{cr} c^2}. \quad (7)$$

In these equations, $\mathbf{p}_{cr} = \gamma_{cr} m_{cr} \mathbf{v}_{cr}$ is the momentum of a cosmic-ray particle having the rest mass m_{cr} and velocity \mathbf{v}_{cr} , q_{cr} is its charge, $p_{cr} = \gamma_{cr}^{-1} n_{cr} T_{cr}$ is the kinetic pressure, n_{cr} is the number density in the laboratory frame, Γ_{cr} is the adiabatic index, $\gamma_{cr} = (1 - \mathbf{v}_{cr}^2/c^2)^{-1/2}$ is the relativistic factor. The continuity equation for cosmic rays is the same as Equation (2) at $j = cr$. Equation (7) can be used for both cold nonrelativistic, $T_{cr} \ll m_{cr} c^2$, and hot relativistic, $T_{cr} \gg m_{cr} c^2$, cosmic rays. In the first (second) case, we have $\Gamma_{cr} = 5/3$ ($4/3$) (Lontano et al. 2001). The general form of the value R_{cr} , which is valid for any relations between T_{cr} and $m_{cr} c^2$, can be found, e.g., in Toepfer (1971) and Dzhavakhishvili & Tsintsadze (1973). We note that relativistic MHD equations are obtained from the kinetic equations for species (e.g., Toepfer 1971; Dzhavakhishvili & Tsintsadze 1973) and the form of Equations (5)-(7) is equivalent to equations for cosmic rays used in other papers (e.g., Sakai & Kawata 1980; Mikhailovskii et al. 1985; Mofiz & Khan 1993; Gratton et al. 1998; Haim 2009). It should be noted that in general the notation T_{cr} is considered not to be as the temperature, but as some typical internal energy of the cosmic-ray distribution. To avoid confusion, this notation could be changed via p_{cr} . However, we retain it as it is given in (Lontano et al. 2001). The relativistic MHD equation (5) has a general form and can be also applied to nonrelativistic and relativistic fluid flows or beam particles (see, e.g., Toepfer 1971; Wallis et al. 1975; Hazeltine & Mahajan 2000; Haim 2009). We note that in the multifluid part of their paper (Appendix A), Riquelme & Spitkovsky (2010) have used the equation for cosmic rays (Equation (A1)) analogous to Equation (5) in the cold temperature

regime with a beam velocity. We also note that the simple one-fluid MHD equations have been used by Sharma et al. (2009, 2010) to consider the influence of adiabatic cosmic rays with the diffusive energy flux on the buoyancy and thermal instabilities in galaxy clusters, correspondingly.

Equations (1)-(6) are solved together with Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad (8)$$

and

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (9)$$

where $\mathbf{j} = \mathbf{j}_{pl} + \mathbf{j}_{cr} = \sum_j q_j n_j \mathbf{v}_j + \mathbf{j}_{cr}$. We note that Gauss' law for \mathbf{B} is automatically followed from Equation (8), and Gauss' law for \mathbf{E} is automatically obtained from Equations (2) and (9).

3. Zero order system state

It is known for a long time that a return current is induced in a plasma penetrated by an external beam current (Roberts & Bennett 1968). The return plasma current equal to the external one and directed oppositely arises due to self-consistent electromagnetic perturbations of plasma under the action of an external current (e.g., Cox & Bennett 1970; Hammer & Rostoker 1970; Berk & Pearlstein 1976). As a result, the condition of quasineutrality and the absence of the total current are maintained. In astrophysical plasmas, such external beam currents are cosmic-ray flows. In papers devoted to cosmic-ray streaming instabilities in the situation where drift velocities of plasma species and cosmic rays are directed along the background magnetic field (e.g., Achterberg 1983; Zweibel 2003; Bell 2004; Riquelmi & Spitkovsky 2009; Nekrasov 2013), it has been also assumed

that to zeroth-order the system is of charge neutrality and there is no net current due to appearance of the plasma return current. Here, we consider other situation in which cosmic rays can drift across the background magnetic field. One such a possibility has been considered by Riquelme & Spitkovsky (2010) for the upstream medium of supernova remnant shocks. It was shown that near the shock cosmic rays having the Larmor radius smaller than the length scale of pre-amplified, quasi-transverse magnetic field generated by the highest energy cosmic rays due to the Bell instability (Bell 2004), will produce a current perpendicular to the initial, pre-amplified field due to the coherent deflection in the “homogeneous” (large scale) magnetic field (see Riquelme & Spitkovsky 2010 for details). One can say that this perpendicular current arises due to the magnetic wall effect. Therefore, we would like to note that such a mechanism could also occur in other astrophysical environments where cosmic rays can encounter magnetic fields (clouds). Two-dimensional particle-in-cell simulations (Riquelme & Spitkovsky 2010) have confirmed the formation of the perpendicular mean cosmic-ray velocity (at $\sim c/2$).

As in the case of cosmic rays drifting along the magnetic field, one can also assume the generation of the return plasma current compensating the perpendicular cosmic-ray one. It can be shown that in the ideal model of Riquelme & Spitkovsky (2010), we have an infinite sheet cosmic-ray current, which forms a homogeneous magnetic field parallel to the current plane and perpendicular to the current direction. In this case, the return current can be only produced by the time-dependent perpendicular electric field in the zero order state, in which plasma species experience a polarization drift across the magnetic field.

Let us find this electric field. We consider a uniform plasma embedded in the uniform magnetic field \mathbf{B}_0 , \mathbf{j}_{cr0} is directed along the y -axis. From Equations (1) and (5), where we take into account the electric and polarization drifts of particles, and from Equation (9) without left-hand side and with account for the displacement current, one can find the

time-dependent zero order electric field \mathbf{E}_0 defined by

$$\frac{\partial \mathbf{E}_0}{\partial t} = -4\pi \mathbf{j}_{cr0} \frac{c_{Ai}^2}{c_{Ai}^2 + c^2}, \quad (10)$$

where $c_{Ai} = (B_0^2/4\pi m_i n_{i0})^{1/2}$ is the ion Alfvén velocity. The conditions $\partial/\partial t \ll \omega_{cj}$ and $R_{cr}\gamma_{cr}\partial/\partial t \ll \omega_{cjr}$, where $\omega_{cj} = q_j B_0/m_j c$ is the cyclotron frequency, and the condition of quasi-neutrality, $q_i n_{i0} + q_e n_{e0} + q_{cr} n_{cr0} = 0$ (the number density n_{cr} is the one in the laboratory frame), have been used. The polarization drift of cosmic rays in Equation (10) has been omitted. This equation in the case $c^2 \gg c_{Ai}^2$ has been given by Riquelme & Spitkovsky (2010) without derivation. We note that at the absence of the background plasma ($n_{i0} \rightarrow 0$, $c_{Ai} \rightarrow \infty$), Equation (10) results in Maxwell's equation $4\pi \mathbf{j}_{cr0} + \partial \mathbf{E}_0/\partial t = 0$ for the uniform magnetic field. Using Equation (10) in the limit $c^2 \gg c_{Ai}^2$, we find the return plasma current \mathbf{j}_{ret} defined by the polarization drift of ions \mathbf{u}_{pl}

$$\mathbf{j}_{ret} = q_i n_{i0} \mathbf{u}_{pl} = -\mathbf{j}_{cr0}, \quad (11)$$

whose magnitude is equal to the cosmic-ray current and has the opposite direction. The polarization drift of electrons is not taken into account because of a small electron mass. In general, the zero order electric field \mathbf{E}_0 cannot operate indefinitely. This field continues only during the action of cosmic rays. If we put, for convenience, $\mathbf{j}_{cr0} = q_{cr} n_{cr0} \mathbf{u}_{cr}$, where \mathbf{u}_{cr} is the velocity of cosmic rays along the y -axis, then we obtain from Equation (11) that $\mathbf{u}_{pl} = -(q_{cr} n_{cr0}/q_i n_{i0}) \mathbf{u}_{cr}$. Thus, $u_{pl} \ll u_{cr}$ because $n_{cr0} \ll n_{i0}$. Below, the plasma drift velocity \mathbf{u}_{pl} will be also taken into account together with \mathbf{u}_{cr} .

Above in this Section, we have discussed a zero order state for a model considered by Riquelme & Spitkovsky (2010) in which cosmic-ray and plasma return currents are perpendicular to the background magnetic field. However, perpendicular currents can also form due to other reasons. For example, cosmic rays and plasma charged species can drift across the magnetic field, which is inhomogeneous in the longitudinal and/or transverse

directions, and in the presence of a perpendicular gravitational acceleration. In this case, we think, a return current cannot appear because cosmic rays are not an external agent penetrating a plasma. Further, the large energy cosmic rays having the Larmor radius much larger than inhomogeneities of magnetic force lines can result in a transverse current. It is possible that in this case the return current can arise. Also, diamagnetic drifts due to transverse pressure gradients produce transverse currents.

For simplicity, we further consider the case in which background temperatures of electrons and ions are equal each other, i.e. $T_{e0} = T_{i0} = T_0$. The case $T_{e0} \neq T_{i0}$ for thermal instability has been considered, for instance, by Nekrasov (2011, 2012). Here, we will omit the perturbed terms $\propto (T_{e0} - T_{i0})$ in the temperature equations. However, to follow the symmetric contribution of ions and electrons in a convenient way, we make some calculations by assuming different temperatures. Then, thermal equations (3) and (4) in the background state take the form

$$\mathcal{L}_i(n_{i0}, T_{i0}) = \mathcal{L}_e(n_{e0}, T_{e0}) = 0. \quad (12)$$

4. Wave equations

For perturbations across the background magnetic field when $\partial/\partial z = 0$, Equations (8) and (9) give us the following two equations:

$$c^2 \left(\frac{\partial}{\partial t} \right)^{-2} \left(\frac{\partial^2 E_{1x}}{\partial y^2} - \frac{\partial^2 E_{1y}}{\partial x \partial y} \right) - E_{1x} = 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{1x} \quad (13)$$

and

$$c^2 \left(\frac{\partial}{\partial t} \right)^{-2} \left(-\frac{\partial^2 E_{1x}}{\partial x \partial y} + \frac{\partial^2 E_{1y}}{\partial x^2} \right) - E_{1y} = 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{1y}, \quad (14)$$

where $\mathbf{j}_1 = \mathbf{j}_{pl1} + \mathbf{j}_{cr1}$ and the subscript 1 here and below denotes the perturbed values. The third equation describes the ordinary electromagnetic wave with $\mathbf{E}_1 \parallel \mathbf{B}_0$ and is split from Equations (13) and (14). The general expressions for the components $j_{pl1x,y}$ and $j_{cr1x,y}$ are given in the Appendices A and B (Equations (A54)-(A56) and (B19)-(B21)). These expressions are available for both magnetized and non-magnetized systems, electron-positron, pair-ion and dusty plasmas and so on. Besides, they include the radiation-condensation effects. In their general form, these expressions are very complicated. Therefore to proceed analytically, one must apply simplifying assumptions. We are interested in magnetized systems consisting of electrons, ions, and cosmic rays, in which cyclotron frequencies of species are much larger than the Doppler-shifted dynamical frequencies. In our case, this implies

$$\begin{aligned} \omega_{ci}^2 &\gg \left(\frac{\partial}{\partial t} + u_{pl} \frac{\partial}{\partial y} \right)^2, \\ \omega_{ccr}^2 &\gg \gamma_{cr0}^4 \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^2 \end{aligned} \quad (15)$$

(see Equations (A5), (A8), and (B7)). As we have noted above, cosmic rays can be both protons and electrons. For ultrarelativistic cosmic rays, $\gamma_{cr0} \gg 1$, the second Equation (15) can be violated. Such a case in which cosmic rays become unmagnetized is not considered in this paper. We here assume also that the case $T_{cr} \ll m_{cr} c^2$ is satisfied, i.e. cosmic rays are cold. Another condition that simplifies the treatment considerably is to assume the wavelength of perturbations to be much larger than the thermal Larmor radius of particles ρ_j

$$\begin{aligned} 1 &\gg \rho_i^2 \nabla^2, \\ 1 &\gg \rho_{cr}^2 \gamma_{cr0} \left(\gamma_{cr0}^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right), \end{aligned} \quad (16)$$

where $\rho_i \sim (T_{i0}/m_i \omega_{ci}^2)^{1/2}$ and $\rho_{cr} = c_{scr}/\omega_{ccr}$ (see Equations (A41) and (B11)). The additional conditions for cosmic rays simplifying their contribution to a current will be

given below. The third simplification is to consider perturbations along and across the cosmic-ray velocity \mathbf{u}_{cr} separately. The first case is simpler. Therefore, we begin with its consideration.

5. The case $\frac{\partial}{\partial y} \neq 0, \frac{\partial}{\partial x} = 0$

Using Equation (A56) and performing calculations of the corresponding quantities, we find the components of the plasma dielectric permeability tensor (v_{i0y} has been changed by u_{pl})

$$\begin{aligned}
 \varepsilon_{plx} &= \frac{\omega_{pi}^2}{\omega_{ci}^2} \left(\frac{\partial}{\partial t} + u_{pl} \frac{\partial}{\partial y} \right)^2 \left(\frac{\partial}{\partial t} \right)^{-2} \\
 &\quad - \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{1}{m_i} \left[T_{i0} + T_{e0} - \frac{G_1 + G_3}{D} \frac{\partial}{\partial t} - \frac{G_2 + G_4}{D} \left(\frac{\partial}{\partial t} + u_{pl} \frac{\partial}{\partial y} \right) \right] \frac{\partial^2}{\partial y^2} \left(\frac{\partial}{\partial t} \right)^{-2}, \\
 \varepsilon_{plx} &= \left(\frac{\omega_{pi}^2 \omega_{ci}}{\Omega_i^2} + \frac{\omega_{pe}^2 \omega_{ce}}{\Omega_e^2} \right) \left(\frac{\partial}{\partial t} \right)^{-1} \\
 &\quad + \frac{\omega_{pi}^2}{\omega_{ci}^3} \frac{1}{m_i} \left[T_{i0} - \frac{G_2 + G_4}{D} \left(\frac{\partial}{\partial t} + u_{pl} \frac{\partial}{\partial y} \right) \right] \frac{\partial^2}{\partial y^2} \left(\frac{\partial}{\partial t} \right)^{-1}, \\
 \varepsilon_{ply} &= - \left(\frac{\omega_{pi}^2 \omega_{ci}}{\Omega_i^2} + \frac{\omega_{pe}^2 \omega_{ce}}{\Omega_e^2} \right) \left(\frac{\partial}{\partial t} \right)^{-1} \\
 &\quad - \frac{\omega_{pi}^2}{\omega_{ci}^3} \frac{1}{m_i} \left[T_{i0} - \frac{G_3}{D} \frac{\partial}{\partial t} - \frac{G_4}{D} \left(\frac{\partial}{\partial t} + u_{pl} \frac{\partial}{\partial y} \right) \right] \frac{\partial^2}{\partial y^2} \left(\frac{\partial}{\partial t} \right)^{-1}, \\
 \varepsilon_{ply} &= \frac{\omega_{pi}^2}{\omega_{ci}^2}.
 \end{aligned} \tag{17}$$

For obtaining Equation (17), we have taken into account that $m_i \gg m_e$ and $n_{i0} \simeq n_{e0}$.

Analogously from Equation (B21), we obtain the cosmic-ray dielectric permeability tensor

$$\begin{aligned}
\varepsilon_{crxx} &= \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^3 \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^2 \left(\frac{\partial}{\partial t} \right)^{-2} - \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^2 c_{scr}^2 \left(\frac{u_{cr}}{c^2} \frac{\partial}{\partial t} + \frac{\partial}{\partial y} \right) \frac{\partial}{\partial y} \left(\frac{\partial}{\partial t} \right)^{-2}, \quad (18) \\
\varepsilon_{crxy} &= -\varepsilon_{cryx} = \frac{\omega_{pcr}^2}{\Omega_{cr}^2} \omega_{ccr} \left(\frac{\partial}{\partial t} \right)^{-1} + \frac{\omega_{pcr}^2}{\omega_{ccr}^3} \gamma_{cr0}^3 c_{scr}^2 \left(\frac{u_{cr}}{c^2} \frac{\partial}{\partial t} + \frac{\partial}{\partial y} \right) \frac{\partial}{\partial y} \left(\frac{\partial}{\partial t} \right)^{-1}, \\
\varepsilon_{cryy} &= \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}.
\end{aligned}$$

Here, we have used the additional condition for cosmic rays

$$1 \gg \gamma_{cr0}^3 \rho_{cr}^2 \frac{u_{cr}}{c^2} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \frac{\partial}{\partial y}$$

(see Equation (B11)). The term proportional to u_{cr}/c^2 in Equation (18) shows the contribution of γ_{cr1} to the cosmic-ray pressure perturbation (see Equations (B8) and (B9)).

5.1. Wave equation

From Equations (13) and (14), using Equations (A54), (A55), (B19), and (B20) and omitting the contribution of the displacement current under condition $\varepsilon_{xx} \gg 1$, we obtain equation

$$\varepsilon_{yy} c^2 \left(\frac{\partial}{\partial t} \right)^{-2} \frac{\partial^2 E_{1x}}{\partial y^2} = (\varepsilon_{xx} \varepsilon_{yy} - \varepsilon_{xy} \varepsilon_{yx}) E_{1x}, \quad (19)$$

where $\varepsilon_{ij} = \varepsilon_{plij} + \varepsilon_{crij}$. The values ε_{ij} are defined by Equations (17) and (18). When calculating the right-hand side of Equation (19), we assume some additional conditions except those given by Equations (15) and (16). We will neglect the contribution to $\varepsilon_{xy} \varepsilon_{yx}$ of the thermal cosmic-ray term in ε_{crxy} and ε_{cryx} . Besides, we will use the condition of quasineutrality in ε_{xy} and ε_{yx} and neglect the terms arising due to expansion of $\Omega_{i,cr}^{-2}$. An

analysis shows that the corresponding conditions can be written in the form

$$\begin{aligned} & \min \left\{ \gamma_{cr0} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^2 ; c_{scr}^2 \left(\frac{u_{cr}}{c^2} \frac{\partial}{\partial t} + \frac{\partial}{\partial y} \right) \frac{\partial}{\partial y} \right\} \\ & \gg \gamma_{cr0}^3 \frac{c_{scr}^4}{\omega_{ccr}^2} \left(\frac{u_{cr}}{c^2} \frac{\partial}{\partial t} + \frac{\partial}{\partial y} \right)^2 \frac{\partial^2}{\partial y^2} ; \gamma_{cr0} \frac{c_{spl}^2 c_{scr}^2}{\omega_{ci} \omega_{ccr}} \left(\frac{u_{cr}}{c^2} \frac{\partial}{\partial t} + \frac{\partial}{\partial y} \right) \frac{\partial^3}{\partial y^3} ; \\ & \gamma_{cr0} \frac{c_{scr}^2}{\omega_{ci} \omega_{ccr}} \left(\frac{u_{cr}}{c^2} \frac{\partial}{\partial t} + \frac{\partial}{\partial y} \right) \frac{\partial}{\partial y} \left(\frac{\partial}{\partial t} + u_{pl} \frac{\partial}{\partial y} \right)^2 , \end{aligned} \quad (20)$$

where $c_{spl} = (2\gamma T_{i0}/m_i)^{1/2}$. For simplicity, for writing these inequalities, we considered the terms, in which plasma frequencies ω_{pi} and ω_{pcr} are cancelled. In the term ε_{xx} , we used a cosmic-ray term in the main. According to Equations (16) and (20), the contribution of the term $\varepsilon_{xy}\varepsilon_{yx}$ to Equation (19) is small. For example, an estimation shows (without thermal terms in Equation (18)) that $\varepsilon_{plx}\varepsilon_{ply}/\varepsilon_{plx}\varepsilon_{plyy} \sim (\partial/\partial t + u_{pl}\partial/\partial y)^2/\omega_{ci}^2 \ll 1$ and $\varepsilon_{crxy}\varepsilon_{cryx}/\varepsilon_{crxx}\varepsilon_{cryy} \sim (\partial/\partial t + u_{cr}\partial/\partial y)^2/\gamma_{cr}^4\omega_{ccr}^2 \ll 1$ (see Equation (15)). Thus, we obtain the simple wave equation

$$c^2 \frac{\partial^2 E_{1x}}{\partial y^2} = \varepsilon_{xx} \left(\frac{\partial}{\partial t} \right)^2 E_{1x}. \quad (21)$$

We note that for these perturbations $E_{1y} = 0$ and $B_{1x,y} = 0$, $B_{1z} \neq 0$.

5.2. Dispersion relation

Using Equations (17) and (18) to find ε_{xx} and accomplishing the Fourier transform in Equation (21), we find for perturbations of the form $\exp(ik_y y - i\omega t)$ the following dispersion relation:

$$\begin{aligned} 0 = & \frac{\omega_{pi}^2}{\omega_{ci}^2} (\omega - k_y u_{pl})^2 + \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^3 (\omega - k_y u_{cr})^2 \\ & - \frac{\omega_{pi}^2}{\omega_{ci}^2} k_y^2 \frac{1}{m_i} \left[T_{i0} + T_{e0} + \frac{G_1 + G_3}{D} i\omega + \frac{G_2 + G_4}{D} i(\omega - k_y u_{pl}) \right] \\ & - \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^2 k_y^2 c_{scr}^2 - k_y^2 c^2. \end{aligned} \quad (22)$$

Below, we consider solutions of Equation (22) for the streaming instability and an influence of the streaming and thermal pressure effects on the thermal instability.

5.2.1. Streaming instability

Let us set in Equation (22) all frequencies Ω equal to zero. To be more specific, it means that $\omega - k_y u_{pl} \gg \Omega_{T,ni}, \Omega_\epsilon$ and $\omega \gg \Omega_{T,ne}, \Omega_\epsilon$, where $\Omega_{ie} \simeq \Omega_{ei} = \Omega_\epsilon$ (the frequencies Ω are defined by Equation (A12)). These conditions mean that we consider perturbations much faster than the typical time scales of thermal instability. Then, this equation takes the form

$$0 = \frac{\omega_{pi}^2}{\omega_{ci}^2} (\omega - k_y u_{pl})^2 + \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^3 (\omega - k_y u_{cr})^2 - \left(\frac{\omega_{pi}^2}{\omega_{ci}^2} c_{spl}^2 + \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^2 c_{scr}^2 + c^2 \right) k_y^2. \quad (23)$$

The solution of Equation (23) is the following:

$$\omega = \frac{k_y (u_{pl} + d u_{cr})}{1 + d} \pm \frac{k_y}{1 + d} \left[- (u_{cr} - u_{pl})^2 d + (1 + d) (c_{spl}^2 + \gamma_{cr0}^{-1} d c_{scr}^2 + c_{Ai}^2) \right]^{1/2}, \quad (24)$$

where

$$d = \frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^3 = \frac{m_{cr}}{m_i} \frac{n_{cr0}}{n_{i0}} \gamma_{cr0}^3. \quad (25)$$

We see that the streaming instability has a threshold u_{crth} defined by the sound and ion Alfvén velocities

$$u_{crth}^2 = (1 + d^{-1}) (c_{spl}^2 + \gamma_{cr0}^{-1} d c_{scr}^2 + c_{Ai}^2). \quad (26)$$

When this threshold is exceeded, $u_{cr}^2 \gg u_{crth}^2$, the growth rate δ_{gr} is given by

$$\delta_{gr} = \frac{d^{1/2}}{1 + d} k_y u_{cr}. \quad (27)$$

These perturbations move with the phase velocity $v_{ph} = (u_{pl} + d u_{cr}) / (1 + d)$. We see that the induced plasma drift velocity u_{pl} does not affect on the growth rate because $u_{pl} \ll u_{cr}$ (see Equation (11)), but can contribute to the real part of the frequency.

5.2.2. Thermal instability

We now take into account the terms describing the thermal instability in Equation (22). We consider the fast thermal energy exchange regime in which $\Omega_e \gg \partial/\partial t, \Omega_{Ti,e}$. Using Equations (A29) and (A30), we have

$$\frac{\gamma(2\omega - k_y u_{pl}) + i\Omega_{T,n}}{\gamma(2\omega - k_y u_{pl}) + i\gamma\Omega_T} = c_{spl}^{-2} \left(du_{cr}^2 - \gamma_{cr0}^{-1} dc_{scr}^2 - c_{Ai}^2 + \frac{\omega^2}{k_y^2} \right), \quad (28)$$

where

$$\begin{aligned} \Omega_{T,n} &= \Omega_{Te} + \Omega_{Ti} - \Omega_{ne} - \Omega_{ni}, \\ \Omega_T &= \Omega_{Te} + \Omega_{Ti}. \end{aligned}$$

When obtaining Equation (28), we have assumed $\omega \ll k_y u_{cr}$ that physically corresponds to the low frequency thermal instability in a comparison roughly with the streaming instability. If the right-hand side of Equation (28) is much less than unity, we obtain Field's isobaric solution $2\omega = k_y u_{pl} - i\Omega_{T,n}/\gamma$ (Field 1965). These perturbations travel with the phase velocity $u_{pl}/2$. In the opposite case, Equation (28) has the Parker's isochoric solution $2\omega = k_y u_{pl} - i\Omega_T$ (Parker 1953). Thus, the presence of streaming cosmic rays can only change the kind of thermal instability, but not influence on its growth rates. When the right-hand side of Equation (28) is of the order of unity, the limiting solutions intermix.

6. The case $\frac{\partial}{\partial x} \neq 0, \frac{\partial}{\partial y} = 0$

Calculating the components of the plasma dielectric permeability tensor given by Equation (A56), we obtain

$$\begin{aligned}
 \varepsilon_{plx} &= \frac{\omega_{pi}^2}{\omega_{ci}^2}, \\
 \varepsilon_{plx} &= \frac{\omega_{pi}^2 \omega_{ci}}{\Omega_i^2} \left(\frac{\partial}{\partial t} \right)^{-1} + \frac{\omega_{pe}^2 \omega_{ce}}{\Omega_e^2} \left(\frac{\partial}{\partial t} \right)^{-1} \\
 &\quad + \frac{\omega_{pi}^2}{\omega_{ci}^3} \left[\frac{1}{m_i} \left(T_{i0} - \frac{G_3 + G_4}{D} \frac{\partial}{\partial t} \right) \frac{\partial}{\partial x} - \omega_{ci} u_{pl} \right] \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} \right)^{-1}, \\
 \varepsilon_{ply} &= -\frac{\omega_{pi}^2 \omega_{ci}}{\Omega_i^2} \left(\frac{\partial}{\partial t} \right)^{-1} - \frac{\omega_{pe}^2 \omega_{ce}}{\Omega_e^2} \left(\frac{\partial}{\partial t} \right)^{-1} \\
 &\quad - \frac{\omega_{pi}^2}{\omega_{ci}^3} \left[\frac{1}{m_i} \left(T_{i0} - \frac{G_2 + G_4}{D} \frac{\partial}{\partial t} \right) \frac{\partial}{\partial x} + \omega_{ci} u_{pl} \right] \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} \right)^{-1}, \\
 \varepsilon_{ply} &= \frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{1}{m_i} \left(T_{i0} + T_{e0} - \frac{G_1 + G_2 + G_3 + G_4}{D} \frac{\partial}{\partial t} \right) \frac{\partial^2}{\partial x^2} \left(\frac{\partial}{\partial t} \right)^{-2} \\
 &\quad + \frac{\omega_{pi}^2}{\omega_{ci}^2} u_{pl}^2 \frac{\partial^2}{\partial x^2} \left(\frac{\partial}{\partial t} \right)^{-2} - \frac{\omega_{pi}^2}{\omega_{ci}^3} u_{pl} \frac{1}{m_i} \frac{G_2 - G_3}{D} \frac{\partial^3}{\partial x^3} \left(\frac{\partial}{\partial t} \right)^{-1}.
 \end{aligned} \tag{29}$$

From Equation (B21) for cosmic rays, we have

$$\begin{aligned}
 \varepsilon_{crx} &= \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^3, \\
 \varepsilon_{crx} &= \frac{\omega_{pcr}^2 \omega_{ccr}}{\Omega_{cr}^2} \left(\frac{\partial}{\partial t} \right)^{-1} + \frac{\omega_{pcr}^2}{\omega_{ccr}^3} \gamma_{cr0}^3 \left(\gamma_{cr0}^2 c_{scr}^2 \frac{\partial}{\partial x} - \omega_{ccr} u_{cr} \right) \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} \right)^{-1}, \\
 \varepsilon_{cry} &= -\frac{\omega_{pcr}^2 \omega_{ccr}}{\Omega_{cr}^2} \left(\frac{\partial}{\partial t} \right)^{-1} - \frac{\omega_{pcr}^2}{\omega_{ccr}^3} \gamma_{cr0}^3 \left(c_{scr}^2 \frac{\partial}{\partial x} + \omega_{ccr} u_{cr} \right) \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} \right)^{-1}, \\
 \varepsilon_{cry} &= \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0} \left[1 + \gamma_{cr0}^2 u_{cr}^2 \frac{\partial^2}{\partial x^2} \left(\frac{\partial}{\partial t} \right)^{-2} \right] - \frac{\omega_{pcr}^2}{\omega_{ccr}^2} c_{scr}^2 \gamma_{cr0}^2 \frac{\partial^2}{\partial x^2} \left(\frac{\partial}{\partial t} \right)^{-2}.
 \end{aligned} \tag{30}$$

In this geometry, the additional simplifying condition for the cosmic-ray contribution except of Equation (16) follows from Equation (B11)

$$1 \gg \gamma_{cr0}^2 \frac{u_{cr} c_{scr}}{c^2} \rho_{cr} \frac{\partial}{\partial x}.$$

We note that the terms $\varepsilon_{plx}(\varepsilon_{ply})$ and $\varepsilon_{crx}(\varepsilon_{cry})$ contain here large terms $\omega_{ci} u_{pl}$ and $\omega_{ccr} u_{cr}$, respectively.

6.1. Wave equation

In the case under consideration, the wave equation has the form

$$\varepsilon_{xx} c^2 \left(\frac{\partial}{\partial t} \right)^{-2} \frac{\partial^2 E_{1y}}{\partial x^2} = (\varepsilon_{xx} \varepsilon_{yy} - \varepsilon_{xy} \varepsilon_{yx}) E_{1y}. \quad (31)$$

Using Equations (29) and (30) and calculating the right-hand side of Equation (31), we find the simple expression for $\varepsilon_{xx} \varepsilon_{yy} - \varepsilon_{xy} \varepsilon_{yx}$

$$\begin{aligned} \varepsilon_{xx} \varepsilon_{yy} - \varepsilon_{xy} \varepsilon_{yx} = & \varepsilon_{xx} \left(\frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0} \right) - \varepsilon_{xx} \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^2 c_{scr}^2 \frac{\partial^2}{\partial x^2} \left(\frac{\partial}{\partial t} \right)^{-2} \\ & - \varepsilon_{xx} \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{1}{m_i} \left(T_{i0} + T_{e0} - \frac{G_1 + G_2 + G_3 + G_4}{D} \frac{\partial}{\partial t} \right) \frac{\partial^2}{\partial x^2} \left(\frac{\partial}{\partial t} \right)^{-2} \\ & + \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^3 (u_{cr} - u_{pl})^2 \frac{\partial^2}{\partial x^2} \left(\frac{\partial}{\partial t} \right)^{-2}. \end{aligned} \quad (32)$$

In these perturbations, we have $E_{1x} = 0$ and $B_{1x,y} = 0$, $B_{1z} \neq 0$.

6.2. Dispersion relation

After Fourier transformation of Equation (31) and substitution of Equation (32), we derive the dispersion relation

$$\begin{aligned} \left(1 + \frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0} \right) \omega^2 = & k_x^2 c_{Ai}^2 + \frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^2 k_x^2 c_{scr}^2 \\ & + k_x^2 \frac{1}{m_i} \left(T_{i0} + T_{e0} + \frac{G_1 + G_2 + G_3 + G_4}{D} i\omega \right) \\ & - \frac{1}{\varepsilon_{xx}} \frac{\omega_{pcr}^2}{\omega_{ccr}^2} \gamma_{cr0}^3 k_x^2 (u_{cr} - u_{pl})^2. \end{aligned} \quad (33)$$

Below, as above, we consider the streaming instability and influence of cosmic rays on the thermal instability.

6.2.1. Streaming instability

As above, we again neglect in the values $G_i, i = 1, 2, 3, 4$, and D all the frequencies Ω . Then, Equation (33) takes the form

$$(1 + \gamma_{cr0}^{-2}d) \frac{\omega^2}{k_x^2} = -\frac{d}{1+d}u_{cr}^2 + c_{spl}^2 + \gamma_{cr0}^{-1}dc_{scr}^2 + c_{Ai}^2, \quad (34)$$

where we have omitted u_{pl} in comparison with u_{cr} . This equation describes an aperiodic instability, if the velocity of cosmic rays exceeds the threshold given by Equation (26). The growth rate δ_{gr} when u_{cr} exceeds u_{crth} is the following:

$$\delta_{gr} = \left[\frac{d}{(1+d)(1 + \gamma_{cr0}^{-2}d)} \right]^{1/2} k_x u_{cr}. \quad (35)$$

6.2.2. Thermal instability

Now, we take into account the contribution into Equation (33) of terms describing the thermal instability in the fast thermal energy exchange regime $\Omega_e \gg \partial/\partial t, \Omega_{Ti,e}$. The dispersion relation becomes

$$\frac{2\gamma\omega + i\Omega_{T,n}}{2\gamma\omega + i\gamma\Omega_T} = c_{spl}^{-2} \left[\frac{d}{1+d}u_{cr}^2 - \gamma_{cr0}^{-1}dc_{scr}^2 - c_{Ai}^2 + (1 + \gamma_{cr0}^{-2}d) \frac{\omega^2}{k_x^2} \right]. \quad (36)$$

This equation is analogous to Equation (28). Depending on whether the right-hand side of Equation (36) is much larger or smaller than unity, we will have the Parker (1953) or the Field (1965) instability.

7. Discussion and implications

We first discuss cosmic-ray streaming instabilities found above, which are a powerful source of magnetic amplification. The growth rates given by Equations (27) and (35) have

in somewhat a similar form and increase with decreasing of the perturbation wavelength. The thresholds for the cases $k_x = 0, k_y \neq 0$ and $k_x \neq 0, k_y = 0$ are equal each other (see Equations (24) at $u_{cr} \gg u_{pl}$ and (34)). Thus, streaming cosmic rays generate perturbations in all directions across the ambient magnetic field. However, in the case of strongly relativistic cosmic rays when $\gamma_{cr0}^{-2}d \gg 1$ (the value d is defined by Equation (25)), the growth rate given by Equation (35) is $\gamma_{cr0} \gg 1$ times larger than that described by Equation (27) (for $k_x \sim k_y$). A spectrum of perturbations in the \mathbf{k} -space is limited from above by Equations (15) and (16) and additional conditions (see inequalities after Equations (18) and (30)). For the case $k_x = 0, k_y \neq 0$, Equation (15) of magnetization can be written in the "soft" form

$$\left(\frac{\lambda_y}{2\pi}\right)^2 \gtrsim \max \left\{ \frac{d}{1+d} \frac{u_{cr}^2}{\omega_{ci}^2}, \frac{\gamma_{cr0}^4}{1+d} \frac{u_{cr}^2}{\omega_{ccr}^2} \right\}, \quad (37)$$

where λ_y (λ_x below) is the wavelength along the $y(x)$ -direction. We have assumed that the threshold of instability is exceeded. Equation (16) is the following: $1 \gg k_y^2 \rho_i^2, \gamma_{cr0} k_y^2 \rho_{ccr}^2$.

The "soft" Equation (15) for the case $k_x \neq 0, k_y = 0$ is given by

$$\left(\frac{\lambda_x}{2\pi}\right)^2 \gtrsim \max \left\{ \frac{d}{(1+d)(1+\gamma_{cr0}^{-2}d)} \frac{u_{cr}^2}{\omega_{ci}^2}, \frac{\gamma_{cr0}^4 d}{(1+d)(1+\gamma_{cr0}^{-2}d)} \frac{u_{cr}^2}{\omega_{ccr}^2} \right\}. \quad (38)$$

Equation (16) has the form $1 \gg k_x^2 \rho_i^2, \gamma_{cr0}^3 k_x^2 \rho_{ccr}^2$. Inequalities after Equations (18) and (30) are satisfied. We see that the dependence of the right-hand sides of Equations (37) and (38) on d is different.

From Equations (28) and (36), it is followed that the relations between magnetohydrodynamical parameters of thermal plasma and cosmic rays and the perturbation wavelength determine the kind of thermal instability ranging from the Parker (1953) to the Field (1965) type instability. Thus in our model, the presence of streaming cosmic rays can only change the kind of thermal instability, but not influence on its growth rates. This conclusion is analogous to that in (Nekrasov & Shadmehri 2012). However, the right-hand sides of Equations (28) and (36) and in the corresponding equations of (Nekrasov & Shadmehri

2012) are quite different.

Let us now compare the growth rate for the streaming instability along the background magnetic field found by Nekrasov & Shadmehri (2012) with the growth rates obtained in this paper. The growth rates given by Equations (27) and (35) are of the same order of magnitude, if $\gamma_{cr0} \sim 1$ or $\gamma_{cr0} \gg 1$ and $d \lesssim 1$ (for the same wave numbers). In the case $\gamma_{cr0} \gg 1$ and $d \gg 1$, the growth rate (35) is larger. Therefore, we use Equation (35) for a comparison. The maximal growth rate in (Nekrasov & Shadmehri 2012) is equal to

$$\delta_m = 2j_{cr0} \left(\frac{\pi}{m_{cr} n_{cr0} c^2} \right)^{1/2} \left(\frac{\gamma_{cr0}^{-1} c_A^2}{\gamma_{cr0}^{-1} c_{scr}^2 + c_A^2} \right)^{1/2},$$

where $c_A = c_{Ai} (1 + \gamma_{cr0}^{-2} d)^{-1/2}$. The ratio of this growth rate to the growth rate (35) for the same cosmic-ray drift velocities is the following:

$$\frac{\delta_m}{\delta_{gr}} = (1 + d^{-1})^{1/2} \frac{c_{Ai}}{(c_{scr}^2 + \gamma_{cr0} c_A^2)^{1/2}} \frac{\omega_{pcr}}{k_x c}. \quad (39)$$

Let cosmic rays be the protons. We estimate $k_x = k_{x \max}$ from Equation (38)

$$k_{x \max} \approx \gamma_{cr0}^{-2} (1 + d^{-1})^{1/2} (1 + \gamma_{cr0}^{-2} d)^{1/2} \frac{\omega_{ccr}}{u_{cr}}.$$

Substituting this estimation into Equation (39) for the case $\gamma_{cr0} c_A^2 \gtrsim c_{scr}^2$, we obtain

$$\frac{\delta_m}{\delta_{gr}} \approx \gamma_{cr0}^{3/2} \left(\frac{n_{cr0}}{n_{i0}} \right)^{1/2} \frac{u_{cr}}{c_{Ai}}. \quad (40)$$

Depending on parameters u_{cr} , n_{cr0} and B_0 , this relation can be both less and larger than unity. In the opposite case, $\gamma_{cr0} c_A^2 \ll c_{scr}^2$, Equation (39) takes the form

$$\frac{\delta_m}{\delta_{gr}} \approx \gamma_{cr0}^{3/2} \left(\frac{n_{cr0}}{n_{i0}} \right)^{1/2} \frac{u_{cr}}{c_{Ai}} \frac{\gamma_{cr0}^{1/2} c_A}{c_{scr}}. \quad (41)$$

We see that in this case the right-hand side of Equation (41) is smaller than that of Equation (40). Thus, transverse streaming instabilities induced by the cosmic-ray back-reaction can considerably contribute to turbulence of astrophysical objects and amplification of magnetic fields.

We now consider some specific values of the growth rates (27) and (35) for cosmic-ray protons in galaxy clusters. For the ICM, we take $T_0 = 3$ keV and $B_0 = 1$ μ G. Then, we obtain $\omega_{ci} \simeq 10^{-2}$ s $^{-1}$ and $c_{spl} = 10^8$ cm s $^{-1}$. In the case of weakly relativistic cosmic rays, $u_{cr} \sim c$ and $\gamma_{cr0} \sim 1$, the parameter $d \sim n_{cr0}/n_{i0} \ll 1$. Since $u_{cr} \gg c_{spl}$, the wave number k_y is less than $\omega_{ci}/u_{cr} \sim 3.3 \times 10^{-13}$ cm $^{-1}$ (see Equation (37)). In the real case in which $n_{cr0}/n_{i0} \ll c_{spl}^2/u_{cr}^2$, or $n_{cr0}/n_{i0} \ll 10^{-5}$, the wave number k_x is limited from above by ω_{ci}/c_{spl} , or 10^{-10} cm $^{-1}$. Thus, the upper estimations of the growth rates (27) and (35) are $\delta_{gr} \sim \omega_{ci} (n_{cr0}/n_{i0})^{1/2}$ and $\delta_{gr} = \omega_{ci} (n_{cr0}/n_{i0})^{1/2} (u_{cr}/c_{spl})$, respectively. These values are considerably larger than the Bell instability (Bell 2004). In the case of ultrarelativistic cosmic rays when $d \gg 1$ but $\gamma_{cr0}^{-2}d \ll 1$, or $\gamma_{cr0}^{-3} \ll n_{cr0}/n_{i0} \ll \gamma_{cr0}^{-1}$, we obtain $k_y \lesssim (n_{cr0}/\gamma_{cr0}n_{i0})^{1/2} (\omega_{ci}/u_{cr})$ and $k_x \lesssim \omega_{ci}/\gamma_{cr0}^2 u_{cr}$ (see Equations (37) and (38)). Correspondingly, the limiting growth rates (27) and (35) are the same and equal to $\delta_{gr} \sim \gamma_{cr0}^{-2} \omega_{ci}$. We note that the last expression is independent from the density of cosmic rays. In the case $\gamma_{cr0}^{-2}d \gg 1$, or $n_{cr0}/n_{i0} \gg \gamma_{cr0}^{-1}$, the region of wavelengths of unstable perturbations in the y -direction and the growth rate remain the same as for the case $\gamma_{cr0}^{-2}d \ll 1$. The wave numbers of the x -perturbations satisfy to $k_x \lesssim (n_{cr0}/\gamma_{cr0}^3 n_{i0})^{1/2} (\omega_{ci}/u_{cr})$ and the corresponding growth rate is equal to $\delta_{gr} \sim \gamma_{cr0}^{-2} \omega_{ci}$ as above. In the case $d \sim 1$, or $n_{cr0}/n_{i0} \sim \gamma_{cr0}^{-3}$, the growth rates are the same in both cases and are equal to the last expression.

In Section 3, we have marked some other mechanisms of the appearance of perpendicular currents except of the model by Riquelme & Spitkovsky (2010). In general, the results for new instabilities obtained here do not change because we use in the equations of motion for species the cosmic-ray and plasma drift velocities \mathbf{u}_{cr} and \mathbf{u}_{pl} , which are not specified for concrete mechanisms. The contribution of electron drifts will be negligible. The specific forms of values \mathbf{u}_{cr} and \mathbf{u}_{pl} will depend on the origin of the perpendicular current. In the case $\mathbf{u}_{cr} \gg \mathbf{u}_{pl}$, the return current of a plasma in dispersion relations (22) and (33) is not

important.

In this paper, we for simplicity did not take into account the electron-ion collisions in the momentum equation (1). For perturbations across the magnetic field, the condition allowing to neglect this effect can be found in (Nekrasov 2012, Equation (34)) and has the form

$$1 \gg \frac{\nu_{ie}\omega}{\omega_{ci}^2} \left(1 + \frac{k_{x,y}^2 c_{spl}^2}{\omega^2} \right).$$

The presence of the ion drift velocity u_{pl} resulting in the Doppler shift (see Appendix A) does not influence on this condition because $\omega \gg k_y u_{pl}$ for the streaming instability and $\omega \gtrsim k_y u_{pl}$ for the thermal one (see Section 5.2). However, the collision frequency in the energy equation, $\Omega_\epsilon = 2\nu_{ie}$, is added to frequencies $\partial/\partial t$ and $\Omega_{Ti,e}$ (see Equations (A29) and (A30)). Thus, the contributions to the dispersion relation of collisions in the momentum equation and in the energy equation are quite different. Therefore, the collisional energy exchange between electrons and ions is included in our analysis.

The model explored here with cosmic rays propagating across the ambient magnetic field has been considered by Bell (2005) (a general case) and have been applied by Riquelme & Spitkovsky (2010) for the problem of the magnetic field amplification in the upstream region of the supernova remnant shocks. However, streaming cosmic-ray driven instabilities can exist in a variety of environments. Therefore, we believe, wherever there is a cosmic-ray streaming, these instabilities may play a significant role. For example, the model described above can be applied to the ICM where cosmic rays are an important ingredient (Loewenstein et al. 1991; Guo & Oh 2008; Sharma et al. 2009; Sharma et al. 2010). Observations show that many cavities or bubbles in the ICM contain cosmic rays and magnetic fields (e.g., Guo & Oh 2008). A substantial amount of cosmic rays may escape from these buoyantly rising bubbles (Enßlin 2003), which could be disrupted by the Rayleigh-Taylor and Kelvin-Helmholtz instabilities as they rise through the ICM (Fabian

et al. 2006). Cosmic rays may also be produced by other processes near a central active galactic nucleus of the galaxy cluster. Structure formation shocks, merger shocks and supernovae may also inject cosmic rays into the ICM (e.g., Voelk et al. 1996; Berezhinsky et al. 1997). Thus, various cosmic-ray streaming instabilities considered in particular in this paper can be a powerful source of generation of magnetic fields in astrophysical settings.

8. Conclusion

We have investigated streaming and thermal instabilities of astrophysical plasmas consisting of electrons, ions, and cosmic rays propagating across the background magnetic field. The drift velocity of cosmic rays can be relativistic, however their mean energy is assumed to be small (non-relativistic). The return current of the background plasma and the back-reaction of magnetized cosmic rays are taken into account. We have considered perturbations which are transverse to the background magnetic field and are along and across the cosmic-ray drift velocity. The case of perturbations along the magnetic field was treated by Nekrasov & Shadmehri (2012) where the growth rate due to the back-reaction of cosmic rays considerably larger than that obtained by Bell (2004, 2005) and Riquelme & Spitkovsky (2010) has been found. In the present case, we have shown that for sufficiently short-wavelength perturbations the growth rates obtained can in their turn exceed the growth rate found in (Nekrasov & Shadmehri 2012). This new result increases the role of cosmic-ray streaming instabilities in amplification of magnetic fields in astrophysical environments.

We have found that the thermal instability is not subject to the action of cosmic rays in the model under consideration. The dispersion relations derived for thermal instability include sound velocities of plasma and cosmic rays, Alfvén and cosmic-ray drift velocities.

The relations between these parameters determine the kind of thermal instability ranging from the Parker (1953) to the Field (1965) type instability. However, the growth rates of thermal instabilities do not change.

The results of this paper can be applied to investigations of weakly collisional electron-ion astrophysical objects such as supernova remnant shocks, galaxy clusters and others, which include the dynamics of streaming cosmic rays.

References

- Achterberg, A. 1983, *A&A*, 119, 274
- Balbus, S. A., & Soker, N. 1989, *ApJ*, 341, 611
- Begelman, M. C., & McKee, C. F. 1990, *ApJ*, 358, 375
- Begelman, M. C., & Zweibel, E. G. 1994, *ApJ*, 431, 689
- Bell, A. R. 2004, *MNRAS*, 353, 550
- Bell, A. R. 2005, *MNRAS*, 358, 181
- Berezinsky, V. S., Blasi, P., & Ptuskin, V. S. 1997, *ApJ*, 487, 529
- Beresnyak, A., Jones, T. W., & Lazarian, A. 2009, *ApJ*, 707, 1541
- Berk, H. L., & Pearlstein, L. D. 1976, *Phys. Fluids*, 19, 1831
- Bogdanović, T., Reynolds, C. S., Balbus, S. A., & Parrish, I. J. 2009, *ApJ*, 704, 211
- Braginskii, S. I. 1965, *Rev. Plasma Phys.*, 1, 205
- Cavagnolo, K. W., Donahue, M., Voil, G. M., & Sun, M. 2008, *ApJ*, 683, L107

- Conselice, C. J., Gallagher, J. S., III, & Wyse, R. F. G. 2001, *AJ*, 122, 2281
- Cox, J. L. Jr, & Bennett, W. H. 1970, *Phys. Fluids*, 13, 182
- Dzhavakhishvili, D. I., & Tsintsadze, N. L. 1973, *Sov. Phys. JEPT*, 37, 666
- Enßlin, T. A. 2003, *A&A*, 399, 409
- Enßlin, T., Pfrommer, C., Miniati, F., & Subramanian, K. 2011, *A&A*, 527, A99
- Everett, J. E., Zweibel, E. G., Benjamin, R. A., McCammon, D., Rocks, L., & Gallagher, J. S., III. 2008, *ApJ*, 674, 258
- Fabian, A. C., Sanders, J. S., Taylor, G. B., Allen, S. W., Crawford, C. S., Johnstone, R. M., & Iwasawa, K. 2006, *MNRAS*, 366, 417
- Ferland, G. J., Fabian, A. C., Hatch, N. A., Johnstone, R. M., Porter, R. L., van Hoof, P. A. M., & Williams, R. J. R. 2009, *MNRAS*, 392, 1475
- Field, G. B. 1965, *ApJ*, 142, 531
- Field, G. B., Goldsmith, D. W., & Habing, H. J. 1969, *ApJ*, 155, L149
- Fukue, T., & Kamaya, H. 2007, *ApJ*, 669, 363
- Gammie, C. F. 1996, *ApJ*, 457, 355
- Gratton, F. T., Gnavi, G., Galvão, R. M. O., & Gomberooff, L. 1998, *Ap&SS*, 256, 311
- Guo, F., & Oh, S. P. 2008, *MNRAS*, 384, 251
- Haim, L. 2009, Nonlinear waves and shocks in relativistic two-fluid hydrodynamics, *Thesis for the degree of master of science*, Department of Physics, Faculty of Natural Sciences, Ben-Gurion University of the Negev
- Hammer, D. A., & Rostoker, N. 1970, *Phys. Fluids*, 13, 1831

- Hazeltine, R. D., & Mahajan, S. M. 2000, Relativistic magnetohydrodynamics, Institute for Fusion Studies (Austin: Univ. of Texas); http://w3fusion.ph.utexas.edu/ifs/ifsreports/913_hazeltine.pdf
- Hennebelle, P., & Pérouault, M. 2000, *A&A*, 359, 1124
- Inoue, T., & Inutsuka, S. 2008, *ApJ*, 687, 303
- Koyama, H., & Inutsuka, S. 2000, *ApJ*, 532, 980
- Kulsrud, R., & Pearce, W. P. 1969, *ApJ*, 156, 445
- Loewenstein, M. 1990, *ApJ*, 349, 471
- Loewenstein, M., Zweibel, E. G., & Begelman, M. C. 1991, *ApJ*, 377, 392
- Lontano, M., Bulanov, S., & Koga, J. 2001, *Phys. Plasmas*, 8, 5113
- Mathews, W., & Bregman, J. 1978, *ApJ*, 224, 308
- Mikhailovskii, A. B., Onischenko, O. G., & Tatarinov, E. G. 1985, *Plasma Phys. Control. Fusion*, 27, 527
- Mofiz, U. A., & Khan, F. 1993, *Ap&SS*, 201, 53
- Nekrasov, A. K. 2007, *Phys. Plasmas*, 14, 062107
- Nekrasov, A. K. 2011, *ApJ*, 739, A88
- Nekrasov, A. K. 2012, *MNRAS*, 419, 522
- Nekrasov, A. K. 2013, *Plasma Phys. Control. Fusion*, 55, 085007
- Nekrasov, A. K., & Shadmehri, M. 2012, *ApJ*, 756, A77
- O’Dea, C. P., Baum, S. A., Privon, G., et al. 2008, *ApJ*, 681, 1035
- Parker, E. N. 1953, *ApJ*, 117, 431

- Parrish, I. J., Quataert, E., & Sharma, P. 2009, *ApJ*, 703, 96
- Riquelme, M. A., & Spitkovsky, A. 2009, *ApJ*, 694, 626
- Riquelme, M. A., & Spitkovsky, A. 2010, *ApJ*, 717, 1054
- Roberts, T. G., & Bennett, W. H. 1968, *Plasma Phys.*, 10, 381
- Sakai, J., & Kawata, T. J. 1980, *Phys. Soc. Japan*, 49, 747
- Salomé, P., Combes, F., Edge, A. C., et al. 2006, *A&A*, 454, 437
- Samui, S., Subramanian, K., & Srianand, R. 2010, *MNRAS*, 402, 2778
- Sánchez-Salcedo, F. J., Vázquez-Semadeni, E., & Gazol, A. 2002, *ApJ*, 577, 768
- Shadmehri, M., Nejad-Asghar, M., & Khesali, A. 2010, *Ap&SS*, 326, 83
- Sharma, P., Chandran, B. D. G., Quataert, E., & Parrish, I. J. 2009, *ApJ*, 699, 348
- Sharma, P., Parrish, I. J., & Quataert, E. 2010, *ApJ*, 720, 652
- Skilling, J. 1975, *MNRAS*, 172, 557
- Toepfer, A. J. 1971, *Phys. Rev. A*, 3, 1444
- Tozzi, P., & Norman, C. 2001, *ApJ*, 546, 63
- Vázquez-Semadeni, E., Ryu, D., Passot, T., González, R. F., & Gazol, A. 2006, *ApJ*, 643, 245
- Voelk, H. J., Aharonian, F. A., & Breitschwerdt, D. 1996, *Space Sci. Rev.*, 75, 279
- Wallis, G., Sauer, K., Sünder, D., Rosinskii, S. E., Rukhadze, A. A., & Rukhlin, V. G. 1975, *Sov. Phys. Usp.*, 17, 492
- Yusef-Zadeh, F., Wardle, M., & Roy, S. 2007, *ApJ*, 665, L123

Zweibel, E. G. 2003, ApJ, 587, 625

Zweibel E. G., & Everett, J. E. 2010, ApJ, 709, 1412

A. Appendix

A.1. Perturbed velocities of ions and electrons

We put in Equation (1) $\mathbf{v}_j = \mathbf{v}_{j0} + \mathbf{v}_{j1}$, $p_j = p_{j0} + p_{j1}$, $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$, $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$, where the subscript 0 denotes equilibrium uniform parameters and the subscript 1 relates to perturbations. Then the linearized version of this equation takes the form

$$\frac{\partial \mathbf{v}_{j1}}{\partial t} + \mathbf{v}_{j0} \cdot \nabla \mathbf{v}_{j1} = -\frac{\nabla T_{j1}}{m_j} - \frac{T_{j0}}{m_j} \frac{\nabla n_{j1}}{n_{j0}} + \mathbf{F}_{j1} + \frac{q_j}{m_j c} \mathbf{v}_{j1} \times \mathbf{B}_0, \quad (\text{A1})$$

where we have used that $p_{j1} = n_{j0} T_{j1} + n_{j1} T_{j0}$ ($n_j = n_{j0} + n_{j1}$, $T_j = T_{j0} + T_{j1}$) and introduced notation

$$\mathbf{F}_{j1} = \frac{q_j}{m_j} \mathbf{E}_1 + \frac{q_j}{m_j c} \mathbf{v}_{j0} \times \mathbf{B}_1. \quad (\text{A2})$$

From Equation (A1), we find expressions for the ion velocities $v_{i1x,y}$ in the form

$$\begin{aligned} \Omega_i^2 v_{i1x} = & \frac{1}{m_i} L_{ix} T_{i1} - \frac{T_{i0}}{m_i} L_{ix} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \nabla \cdot \mathbf{v}_{i1} \\ & + \omega_{ci} F_{i1y} + \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) F_{i1x} \end{aligned} \quad (\text{A3})$$

and

$$\begin{aligned} \Omega_i^2 v_{i1y} = & \frac{1}{m_i} L_{iy} T_{i1} - \frac{T_{i0}}{m_i} L_{iy} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \nabla \cdot \mathbf{v}_{i1} \\ & - \omega_{ci} F_{i1x} + \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) F_{i1y}. \end{aligned} \quad (\text{A4})$$

In Equations (A3) and (A4), we have used the linearized continuity equation (2). The following notations are here introduced:

$$\begin{aligned}\Omega_i^2 &= \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^2 + \omega_{ci}^2, \\ L_{ix} &= -\omega_{ci} \frac{\partial}{\partial y} - \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \frac{\partial}{\partial x}, \\ L_{iy} &= \omega_{ci} \frac{\partial}{\partial x} - \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \frac{\partial}{\partial y}.\end{aligned}\tag{A5}$$

Analogous equations for the electrons are the following:

$$\Omega_e^2 v_{e1x} = \frac{1}{m_e} L_{ex} T_{e1} - \frac{T_{e0}}{m_e} L_{ex} \left(\frac{\partial}{\partial t} \right)^{-1} \nabla \cdot \mathbf{v}_{e1} + \omega_{ce} F_{e1y} + \frac{\partial F_{e1x}}{\partial t},\tag{A6}$$

$$\Omega_e^2 v_{e1y} = \frac{1}{m_e} L_{ey} T_{e1} - \frac{T_{e0}}{m_e} L_{ey} \left(\frac{\partial}{\partial t} \right)^{-1} \nabla \cdot \mathbf{v}_{e1} - \omega_{ce} F_{e1x} + \frac{\partial F_{e1y}}{\partial t},\tag{A7}$$

where

$$\begin{aligned}\Omega_e^2 &= \frac{\partial^2}{\partial t^2} + \omega_{ce}^2, \\ L_{ex} &= -\omega_{ce} \frac{\partial}{\partial y} - \frac{\partial^2}{\partial x \partial t}, \\ L_{ey} &= \omega_{ce} \frac{\partial}{\partial x} - \frac{\partial^2}{\partial y \partial t}.\end{aligned}\tag{A8}$$

We do not consider the longitudinal velocity v_{j1z} because as can be shown in the case $\partial/\partial z = 0$ this velocity only depends on the electric field E_{1z} , $\partial v_{j1z}/\partial t = (q_j/m_j) E_{1z}$, and the transverse and longitudinal wave equations are split.

A.2. Perturbed temperatures of ions and electrons

We find now equations for the temperature perturbations $T_{i,e1}$. We here assume that equilibrium temperatures T_{i0} and T_{e0} are equal one another, $T_{i0} = T_{e0} = T_0$. The case $T_{i0} \neq T_{e0}$ for thermal instability has been considered by Nekrasov (2011, 2012).

For equal temperatures, the terms connected with the perturbation of thermal energy exchange frequency in Equations (3) and (4) will be absent. However for convenience of calculations, we formally retain different notations for the ion and electron temperatures. From Equations (3) and (4) in the linear form, we obtain equations for the temperature perturbations

$$D_{1i}T_{i1} - D_{2i}T_{e1} = C_{1i}\nabla \cdot \mathbf{v}_{i1}, \quad (\text{A9})$$

$$D_{1e}T_{e1} - D_{2e}T_{i1} = C_{1e}\nabla \cdot \mathbf{v}_{e1}, \quad (\text{A10})$$

where notations are introduced

$$\begin{aligned} D_{1i} &= \left[\left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) + \Omega_{Ti} + \Omega_{ie} \right] \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right), \\ D_{2i} &= \Omega_{ie} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right), \\ C_{1i} &= T_{i0} \left[-(\gamma - 1) \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) + \Omega_{ni} \right], \\ D_{1e} &= \left(\frac{\partial}{\partial t} + \Omega_{Te} + \Omega_{ei} \right) \frac{\partial}{\partial t}, \\ D_{2e} &= \Omega_{ei} \frac{\partial}{\partial t}, \\ C_{1e} &= T_{e0} \left[-(\gamma - 1) \frac{\partial}{\partial t} + \Omega_{ne} \right]. \end{aligned} \quad (\text{A11})$$

For obtaining Equations (A9) and (A10), we have used Equations (2) and (12). The frequencies in Equation (A11) are the following:

$$\begin{aligned} \Omega_{Tj} &= (\gamma - 1) \frac{\partial \mathcal{L}_j(n_{j0}, T_{j0})}{n_{j0} \partial T_{j0}}, \Omega_{nj} = (\gamma - 1) \frac{\partial \mathcal{L}_j(n_{j0}, T_{j0})}{T_{j0} \partial n_{j0}}, \\ \Omega_{ie} &= \nu_{ie}^\varepsilon(n_{e0}, T_{e0}), \Omega_{ei} = \nu_{ei}^\varepsilon(n_{i0}, T_{e0}). \end{aligned} \quad (\text{A12})$$

From Equations (A9) and (A10), we find equations for T_{i1} and T_{e1}

$$DT_{i1} = G_4 \nabla \cdot \mathbf{v}_{i1} + G_3 \nabla \cdot \mathbf{v}_{e1} \quad (\text{A13})$$

and

$$DT_{e1} = G_1 \nabla \cdot \mathbf{v}_{e1} + G_2 \nabla \cdot \mathbf{v}_{i1}. \quad (\text{A14})$$

Here, we have

$$\begin{aligned}
D &= D_{1i}D_{1e} - D_{2i}D_{2e}, \\
G_1 &= D_{1i}C_{1e}, G_2 = D_{2e}C_{1i}, \\
G_3 &= D_{2i}C_{1e}, G_4 = D_{1e}C_{1i}.
\end{aligned} \tag{A15}$$

A.3. Expressions for $\nabla \cdot \mathbf{v}_{i,e1}$

We now substitute temperature perturbations $T_{i,e1}$ defined by Equations (A13) and (A14) into Equations (A3) and (A4). Then applying operators $\partial/\partial x$ and $\partial/\partial y$ to Equations (A3) and (A4), respectively, and adding them, we find equation for $\nabla \cdot \mathbf{v}_{i1}$

$$L_{1i}\nabla \cdot \mathbf{v}_{i1} = -L_{2i}\nabla \cdot \mathbf{v}_{e1} + \Phi_{i1}, \tag{A16}$$

where

$$\begin{aligned}
L_{1i} &= \Omega_i^2 + \frac{1}{m_i} \left[\frac{G_4}{D} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) - T_{i0} \right] \nabla^2, \\
L_{2i} &= \frac{1}{m_i} \frac{G_3}{D} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \nabla^2, \\
\Phi_{i1} &= \omega_{ci} \left(\frac{\partial F_{i1y}}{\partial x} - \frac{\partial F_{i1x}}{\partial y} \right) + \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \nabla \cdot \mathbf{F}_{i1}.
\end{aligned} \tag{A17}$$

Analogously, using Equations (A6) and (A7), we obtain

$$L_{1e}\nabla \cdot \mathbf{v}_{e1} = -L_{2e}\nabla \cdot \mathbf{v}_{i1} + \Phi_{e1}, \tag{A18}$$

where

$$\begin{aligned}
L_{1e} &= \Omega_e^2 + \frac{1}{m_e} \left(\frac{G_1}{D} \frac{\partial}{\partial t} - T_{e0} \right) \nabla^2, \\
L_{2e} &= \frac{1}{m_e} \frac{G_2}{D} \frac{\partial}{\partial t} \nabla^2, \\
\Phi_{e1} &= \omega_{ce} \left(\frac{\partial F_{e1y}}{\partial x} - \frac{\partial F_{e1x}}{\partial y} \right) + \frac{\partial}{\partial t} \nabla \cdot \mathbf{F}_{e1}.
\end{aligned} \tag{A19}$$

From Equations (A16) and (A18), we find

$$L\nabla \cdot \mathbf{v}_{i1} = L_{1e}\Phi_{i1} - L_{2i}\Phi_{e1} \quad (\text{A20})$$

and

$$L\nabla \cdot \mathbf{v}_{e1} = L_{1i}\Phi_{e1} - L_{2e}\Phi_{i1}. \quad (\text{A21})$$

The operator L is given by

$$L = L_{1i}L_{1e} - L_{2i}L_{2e}. \quad (\text{A22})$$

A.4. Equations for ion and electron velocities via $\mathbf{F}_{i,e1}$

Using Equations (A3), (A4), (A13), (A20), and (A21), we obtain the following equations for components of the perturbed ion velocity:

$$\Omega_i^2 v_{i1x} = \frac{L_{ix}}{m_i DL} (A_{1i}\Phi_{i1} - A_{2i}\Phi_{e1}) + \omega_{ci} F_{i1y} + \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) F_{i1x} \quad (\text{A23})$$

and

$$\Omega_i^2 v_{i1y} = \frac{L_{iy}}{m_i DL} (A_{1i}\Phi_{i1} - A_{2i}\Phi_{e1}) - \omega_{ci} F_{i1x} + \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) F_{i1y}. \quad (\text{A24})$$

The operators $A_{1,2i}$ are given by

$$\begin{aligned} A_{1i} &= \left[G_4 - DT_{i0} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \right] L_{1e} - G_3 L_{2e}, \\ A_{2i} &= \left[G_4 - DT_{i0} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \right] L_{2i} - G_3 L_{1i}. \end{aligned} \quad (\text{A25})$$

Equations for components of the perturbed electron velocity are found by using Equations (A6), (A7), (A14), (A20), and (A21)

$$\Omega_e^2 v_{e1x} = \frac{L_{ex}}{m_e DL} (A_{1e}\Phi_{e1} - A_{2e}\Phi_{i1}) + \omega_{ce} F_{e1y} + \frac{\partial F_{e1x}}{\partial t}, \quad (\text{A26})$$

$$\Omega_e^2 v_{e1y} = \frac{L_{ey}}{m_e D L} (A_{1e} \Phi_{e1} - A_{2e} \Phi_{i1}) - \omega_{ce} F_{e1x} + \frac{\partial F_{e1y}}{\partial t}. \quad (\text{A27})$$

Here,

$$\begin{aligned} A_{1e} &= \left[G_1 - D T_{e0} \left(\frac{\partial}{\partial t} \right)^{-1} \right] L_{1i} - G_2 L_{2i}, \\ A_{2e} &= \left[G_1 - D T_{e0} \left(\frac{\partial}{\partial t} \right)^{-1} \right] L_{2e} - G_2 L_{1e}. \end{aligned} \quad (\text{A28})$$

A.5. Expressions for D and $G_{1,2,3,4}$

We now give expressions for D and $G_{1,2,3,4}$ defined by Equation (A15). Using Equation (A11), we find

$$\begin{aligned} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \left(\frac{\partial}{\partial t} \right)^{-1} D &= \left[\left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) + \Omega_{Ti} \right] \left(\frac{\partial}{\partial t} + \Omega_{Te} \right) \\ &\quad + \left(\frac{\partial}{\partial t} + \Omega_{Te} \right) \Omega_{ie} + \left[\left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) + \Omega_{Ti} \right] \Omega_{ei} \end{aligned} \quad (\text{A29})$$

and

$$\begin{aligned} G_1 &= T_{e0} \left[\left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) + \Omega_{Ti} + \Omega_{ie} \right] \left[-(\gamma - 1) \frac{\partial}{\partial t} + \Omega_{ne} \right] \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right), \\ G_2 &= T_{i0} \Omega_{ei} \left[-(\gamma - 1) \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) + \Omega_{ni} \right] \frac{\partial}{\partial t}, \\ G_3 &= T_{e0} \Omega_{ie} \left[-(\gamma - 1) \frac{\partial}{\partial t} + \Omega_{ne} \right] \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right), \\ G_4 &= T_{i0} \left(\frac{\partial}{\partial t} + \Omega_{Te} + \Omega_{ei} \right) \left[-(\gamma - 1) \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) + \Omega_{ni} \right] \frac{\partial}{\partial t}. \end{aligned} \quad (\text{A30})$$

A.6. Simplified expressions for $A_{1,2i}$ and $A_{1,2e}$

We can further simplify expressions for $A_{1,2i}$ and $A_{1,2e}$ given by Equations (A25) and (A28). Using Equation (A17), we obtain

$$A_{2i} = -G_3 \Omega_i^2. \quad (\text{A31})$$

The expression for A_{1i} can be given in the form

$$A_{1i} = \left[G_4 - DT_{i0} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \right] \Omega_e^2 - \frac{1}{m_e} \nabla^2 K, \quad (\text{A32})$$

where we have used Equation (A19). The following notation is introduced in Equation (A32):

$$K = \frac{1}{D} (G_2 G_3 - G_1 G_4) \frac{\partial}{\partial t} + G_4 T_{e0} + G_1 T_{i0} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \frac{\partial}{\partial t} - DT_{i0} T_{e0} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1}. \quad (\text{A33})$$

Analogously, we will have

$$A_{2e} = -G_2 \Omega_e^2 \quad (\text{A34})$$

and

$$A_{1e} = \left[G_1 - DT_{e0} \left(\frac{\partial}{\partial t} \right)^{-1} \right] \Omega_i^2 - \frac{1}{m_i} \nabla^2 \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} K. \quad (\text{A35})$$

Calculations show that the value $D^{-1} (G_2 G_3 - G_1 G_4)$ takes the simple form

$$\frac{1}{D} (G_2 G_3 - G_1 G_4) = -T_{i0} T_{e0} \left[-(\gamma - 1) \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) + \Omega_{ni} \right] \left[-(\gamma - 1) \frac{\partial}{\partial t} + \Omega_{ne} \right]. \quad (\text{A36})$$

Using Equations (A29), (A30), and (A36), we can also rewrite the value K defined by Equation (A33) in the simple form

$$K = -T_{i0} T_{e0} (W_i W_e + W_i \Omega_{ei} + W_e \Omega_{ie}) \frac{\partial}{\partial t}. \quad (\text{A37})$$

Here, notations are introduced

$$\begin{aligned} W_i &= \gamma \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) + \Omega_{Ti} - \Omega_{ni}, \\ W_e &= \gamma \frac{\partial}{\partial t} + \Omega_{Te} - \Omega_{ne}. \end{aligned} \quad (\text{A38})$$

We remind the reader that the temperatures of the ions and electrons are considered to be equal one another. We retain different notations for the control of the symmetry of the ion and electron contribution. Analogously, we find the following values:

$$\begin{aligned} G_4 - DT_{i0} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} &= -T_{i0} (W_i V_e + W_i \Omega_{ei} + V_e \Omega_{ie}) \frac{\partial}{\partial t}, \\ G_1 - DT_{e0} \left(\frac{\partial}{\partial t} \right)^{-1} &= -T_{e0} (W_e V_i + W_e \Omega_{ie} + V_i \Omega_{ei}) \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right), \end{aligned} \quad (\text{A39})$$

where

$$\begin{aligned} V_i &= \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) + \Omega_{Ti}, \\ V_e &= \frac{\partial}{\partial t} + \Omega_{Te}. \end{aligned} \quad (\text{A40})$$

A.7. Operator L

Let us find the operator L given by Equation (A22). Using Equations (A17) and (A19), we obtain

$$\begin{aligned} L &= \Omega_i^2 \Omega_e^2 + \frac{1}{m_i} \Omega_e^2 \left[\frac{G_4}{D} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) - T_{i0} \right] \nabla^2 + \frac{1}{m_e} \Omega_i^2 \left(\frac{G_1}{D} \frac{\partial}{\partial t} - T_{e0} \right) \nabla^2 \\ &\quad - \frac{1}{m_i m_e D} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \nabla^4 K. \end{aligned} \quad (\text{A41})$$

The expressions containing in this equation are given by Equations (A37)-(A40).

A.8. Simplified equations for ion and electron velocities via \mathbf{E}_1

We now substitute expressions for $A_{1,2i}$ given by Equations (A31) and (A32) into Equations (A23) and (A24). Then, we replace the values \mathbf{F}_{j1} and $\Phi_{i,e1}$ by their expressions through \mathbf{E}_1 which are given by

$$\begin{aligned} F_{j1x} &= \frac{q_j}{m_j} \left[E_{1x} + v_{j0y} \left(\frac{\partial}{\partial t} \right)^{-1} \left(\frac{\partial E_{1x}}{\partial y} - \frac{\partial E_{1y}}{\partial x} \right) \right], \\ F_{j1y} &= \frac{q_j}{m_j} E_{1y} \end{aligned} \quad (\text{A42})$$

and

$$\begin{aligned} \Phi_{i1} &= -\frac{q_i}{m_i} \left(\omega_{ci} - v_{i0y} \frac{\partial}{\partial x} \right) \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} \left(\frac{\partial E_{1x}}{\partial y} - \frac{\partial E_{1y}}{\partial x} \right) \\ &\quad + \frac{q_i}{m_i} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \nabla \cdot \mathbf{E}_1, \\ \Phi_{e1} &= -\frac{q_e}{m_e} \omega_{ce} \left(\frac{\partial E_{1x}}{\partial y} - \frac{\partial E_{1y}}{\partial x} \right) + \frac{q_e}{m_e} \frac{\partial}{\partial t} \nabla \cdot \mathbf{E}_1. \end{aligned} \quad (\text{A43})$$

For obtaining Equations (A42) and (A43), we have used Equations (A2) and (8). As a result, we will have the following equations for v_{i1x} and v_{i1y} :

$$\begin{aligned} v_{i1x} &= -\frac{q_i}{m_i} \frac{\Omega_e^2}{\Omega_i^2} \frac{L_{ix}}{L} \lambda_i \left[a_i \left(\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} \right) + \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \nabla \cdot \mathbf{E}_1 \right] \\ &\quad + \frac{q_e}{m_e} \frac{L_{ix}}{L} \mu_i \left[\omega_{ce} \left(\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} \right) + \frac{\partial}{\partial t} \nabla \cdot \mathbf{E}_1 \right] \\ &\quad + \frac{q_i}{m_i \Omega_i^2} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^2 \left(\frac{\partial}{\partial t} \right)^{-1} E_{1x} \\ &\quad + \frac{q_i}{m_i \Omega_i^2} \left[\omega_{ci} - v_{i0y} \frac{\partial}{\partial x} \right] \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} E_{1y} \end{aligned} \quad (\text{A44})$$

and

$$\begin{aligned}
v_{i1y} = & -\frac{q_i}{m_i} \frac{\Omega_e^2}{\Omega_i^2} \frac{L_{iy}}{L} \lambda_i \left[a_i \left(\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} \right) + \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \nabla \cdot \mathbf{E}_1 \right] \\
& + \frac{q_e}{m_e} \frac{L_{iy}}{L} \mu_i \left[\omega_{ce} \left(\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} \right) + \frac{\partial}{\partial t} \nabla \cdot \mathbf{E}_1 \right] \\
& - \frac{q_i}{m_i} \frac{\omega_{ci}}{\Omega_i^2} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} E_{1x} \\
& + \frac{q_i}{m_i \Omega_i^2} \left[\omega_{ci} v_{i0y} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} \right)^{-1} + \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \right] E_{1y},
\end{aligned} \tag{A45}$$

where notations are

$$\begin{aligned}
\lambda_i = & \frac{1}{m_i} \left[T_{i0} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} - \frac{G_4}{D} \right] + \frac{1}{m_e m_i D \Omega_e^2} \nabla^2 K, \mu_i = \frac{G_3}{m_i D}, \\
a_i = & \left(\omega_{ci} - v_{i0y} \frac{\partial}{\partial x} \right) \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1}.
\end{aligned} \tag{A46}$$

For the electron velocity, using Equations (A26), (A27), (A34), and (A35), we obtain

$$\begin{aligned}
v_{e1x} = & -\frac{q_e}{m_e} \frac{\Omega_i^2}{\Omega_e^2} \frac{L_{ex}}{L} \lambda_e \left[\omega_{ce} \left(\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} \right) + \frac{\partial}{\partial t} \nabla \cdot \mathbf{E}_1 \right] \\
& + \frac{q_i}{m_i} \frac{L_{ex}}{L} \mu_e \left[a_i \left(\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} \right) + \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \nabla \cdot \mathbf{E}_1 \right] \\
& + \frac{q_e}{m_e} \frac{\omega_{ce}}{\Omega_e^2} E_{1y} + \frac{q_e}{m_e \Omega_e^2} \frac{\partial E_{1x}}{\partial t}
\end{aligned} \tag{A47}$$

and

$$\begin{aligned}
v_{e1y} = & -\frac{q_e}{m_e} \frac{\Omega_i^2}{\Omega_e^2} \frac{L_{ey}}{L} \lambda_e \left[\omega_{ce} \left(\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} \right) + \frac{\partial}{\partial t} \nabla \cdot \mathbf{E}_1 \right] \\
& + \frac{q_i}{m_i} \frac{L_{ey}}{L} \mu_e \left[a_i \left(\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} \right) + \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \nabla \cdot \mathbf{E}_1 \right] \\
& - \frac{q_e}{m_e} \frac{\omega_{ce}}{\Omega_e^2} E_{1x} + \frac{q_e}{m_e \Omega_e^2} \frac{\partial E_{1y}}{\partial t},
\end{aligned} \tag{A48}$$

where

$$\begin{aligned}
\lambda_e = & \frac{1}{m_e} \left[T_{e0} \left(\frac{\partial}{\partial t} \right)^{-1} - \frac{G_1}{D} \right] + \frac{1}{m_i m_e \Omega_i^2 D} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} \nabla^2 K, \\
\mu_e = & \frac{G_2}{m_e D}.
\end{aligned} \tag{A49}$$

A.9. Perturbed plasma currents

We now make use of obtained ion and electron velocities to find perturbed plasma currents $j_{pl1x} = q_i n_{i0} v_{i1x} + q_e n_{e0} v_{e1x}$ and $j_{pl1y} = q_i n_{i0} v_{i1y} + q_i n_{i1} v_{i0y} + q_e n_{e0} v_{e1y}$ in a general form. From Equations (A44) and (A47), we will have

$$\begin{aligned} 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{pl1x} = & \alpha_x \left(\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} \right) - \beta_x \frac{\partial E_{1y}}{\partial x} + \delta_x \nabla \cdot \mathbf{E}_1 \\ & + \frac{\omega_{pi}^2}{\Omega_i^2} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^2 \left(\frac{\partial}{\partial t} \right)^{-2} E_{1x} + \frac{\omega_{pe}^2}{\Omega_e^2} E_{1x} \\ & + \left(\frac{\omega_{pi}^2 \omega_{ci}}{\Omega_i^2} + \frac{\omega_{pe}^2 \omega_{ce}}{\Omega_e^2} \right) \left(\frac{\partial}{\partial t} \right)^{-1} E_{1y}. \end{aligned} \quad (\text{A50})$$

Here,

$$\begin{aligned} \alpha_x = & \frac{1}{L} \left[\omega_{pi}^2 L_{ix} \left(\frac{q_e m_i}{q_i m_e} \mu_i \omega_{ce} - \frac{\Omega_e^2}{\Omega_i^2} \lambda_i a_i \right) + \omega_{pe}^2 L_{ex} \left(\frac{q_i m_e}{q_e m_i} \mu_e a_i - \frac{\Omega_i^2}{\Omega_e^2} \lambda_e \omega_{ce} \right) \right] \left(\frac{\partial}{\partial t} \right)^{-1}, \quad (\text{A51}) \\ \delta_x = & \omega_{pi}^2 \frac{L_{ix}}{L} \left[\frac{q_e m_i}{q_i m_e} \mu_i - \frac{\Omega_e^2}{\Omega_i^2} \lambda_i \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} \right] \\ & + \omega_{pe}^2 \frac{L_{ex}}{L} \left[\frac{q_i m_e}{q_e m_i} \mu_e \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} - \frac{\Omega_i^2}{\Omega_e^2} \lambda_e \right], \\ \beta_x = & \frac{\omega_{pi}^2}{\Omega_i^2} v_{i0y} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-2}, \end{aligned}$$

and $\omega_{pj} = (4\pi n_{j0} q_j^2 / m_j)^{1/2}$ is the plasma frequency. The values $\lambda_{i,e}$, $\mu_{i,e}$, and a_i are given by Equations (A46) and (A49). Using Equations (2), (A44), (A45), and (A48), we further find

$$\begin{aligned} 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{pl1y} = & (\alpha_y + \eta_1) \left(\frac{\partial E_{1y}}{\partial x} - \frac{\partial E_{1x}}{\partial y} \right) - \beta_x \frac{\partial E_{1x}}{\partial x} + \beta_y \frac{\partial E_{1y}}{\partial x} + (\delta_y + \eta_2) \nabla \cdot \mathbf{E}_1 \\ & - \left(\frac{\omega_{pi}^2 \omega_{ci}}{\Omega_i^2} + \frac{\omega_{pe}^2 \omega_{ce}}{\Omega_e^2} \right) \left(\frac{\partial}{\partial t} \right)^{-1} E_{1x} + \left(\frac{\omega_{pi}^2}{\Omega_i^2} + \frac{\omega_{pe}^2}{\Omega_e^2} \right) E_{1y}, \end{aligned} \quad (\text{A52})$$

where

$$\begin{aligned}
\alpha_y &= \omega_{pi}^2 \frac{L_{iy}}{L} \left(\frac{q_e m_i}{q_i m_e} \mu_i \omega_{ce} - \frac{\Omega_e^2}{\Omega_i^2} \lambda_i a_i \right) \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \\
&+ \omega_{pe}^2 \frac{L_{ey}}{L} \left(\frac{q_i m_e}{q_e m_i} \mu_e a_i - \frac{\Omega_i^2}{\Omega_e^2} \lambda_e \omega_{ce} \right) \left(\frac{\partial}{\partial t} \right)^{-1}, \\
\delta_y &= \omega_{pi}^2 \frac{L_{iy}}{L} \left[\frac{q_e m_i}{q_i m_e} \mu_i \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \frac{\partial}{\partial t} - \frac{\Omega_e^2}{\Omega_i^2} \lambda_i \right] \\
&+ \omega_{pe}^2 \frac{L_{ey}}{L} \left[\frac{q_i m_e}{q_e m_i} \mu_e \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} - \frac{\Omega_i^2}{\Omega_e^2} \lambda_e \right], \\
\eta_1 &= \omega_{pi}^2 v_{i0y} \frac{L_{ix}}{L} \left(\frac{\Omega_e^2}{\Omega_i^2} \lambda_i a_i - \frac{q_e m_i}{q_i m_e} \mu_i \omega_{ce} \right) \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \left(\frac{\partial}{\partial t} \right)^{-1} \frac{\partial}{\partial x}, \\
\eta_2 &= \omega_{pi}^2 v_{i0y} \frac{L_{ix}}{L} \left[\frac{\Omega_e^2}{\Omega_i^2} \lambda_i \left(\frac{\partial}{\partial t} \right)^{-1} - \frac{q_e m_i}{q_i m_e} \mu_i \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^{-1} \right] \frac{\partial}{\partial x}, \\
\beta_y &= \frac{\omega_{pi}^2}{\Omega_i^2} v_{i0y}^2 \left(\frac{\partial}{\partial t} \right)^{-2} \frac{\partial}{\partial x}.
\end{aligned} \tag{A53}$$

We can rewrite Equations (A50) and (A52) in the form

$$4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{pl1x} = \varepsilon_{plx} E_{1x} + \varepsilon_{plxy} E_{1y} \tag{A54}$$

and

$$4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{pl1y} = \varepsilon_{ply} E_{1x} + \varepsilon_{plyy} E_{1y}, \tag{A55}$$

where the components of the plasma dielectric permeability tensor are given by

$$\begin{aligned}
\varepsilon_{plx} &= -\alpha_x \frac{\partial}{\partial y} + \delta_x \frac{\partial}{\partial x} + \frac{\omega_{pi}^2}{\Omega_i^2} \left(\frac{\partial}{\partial t} + v_{i0y} \frac{\partial}{\partial y} \right)^2 \left(\frac{\partial}{\partial t} \right)^{-2} + \frac{\omega_{pe}^2}{\Omega_e^2}, \\
\varepsilon_{plxy} &= \alpha_x \frac{\partial}{\partial x} - \beta_x \frac{\partial}{\partial x} + \delta_x \frac{\partial}{\partial y} + \left(\frac{\omega_{pi}^2 \omega_{ci}}{\Omega_i^2} + \frac{\omega_{pe}^2 \omega_{ce}}{\Omega_e^2} \right) \left(\frac{\partial}{\partial t} \right)^{-1}, \\
\varepsilon_{plyx} &= -(\alpha_y + \eta_1) \frac{\partial}{\partial y} - \beta_x \frac{\partial}{\partial x} + (\delta_y + \eta_2) \frac{\partial}{\partial x} - \left(\frac{\omega_{pi}^2 \omega_{ci}}{\Omega_i^2} + \frac{\omega_{pe}^2 \omega_{ce}}{\Omega_e^2} \right) \left(\frac{\partial}{\partial t} \right)^{-1}, \\
\varepsilon_{plyy} &= (\alpha_y + \eta_1) \frac{\partial}{\partial x} + \beta_y \frac{\partial}{\partial x} + (\delta_y + \eta_2) \frac{\partial}{\partial y} + \frac{\omega_{pi}^2}{\Omega_i^2} + \frac{\omega_{pe}^2}{\Omega_e^2}.
\end{aligned} \tag{A56}$$

Using Equations (A51) and (A53), we can find ε_{plij} in specific cases.

B. Appendix

B.1. Perturbed velocity of cosmic rays

The linearized Equation (5) for the cold, nonrelativistic, $T_{cr} \ll m_{cr}c^2$, cosmic rays takes the form

$$\gamma_{cr0} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \left(\mathbf{v}_{cr1} + \gamma_{cr0}^2 \frac{\mathbf{u}_{cr} u_{cr}}{c^2} v_{cr1y} \right) = -\frac{\nabla p_{cr1}}{m_{cr} n_{cr0}} + \mathbf{F}_{cr1} + \frac{q_{cr}}{m_{cr} c} \mathbf{v}_{cr1} \times \mathbf{B}_0, \quad (\text{B1})$$

where

$$\mathbf{F}_{cr1} = \frac{q_{cr}}{m_{cr}} \left(\mathbf{E}_1 + \frac{1}{c} \mathbf{u}_{cr} \times \mathbf{B}_1 \right). \quad (\text{B2})$$

For obtaining Equation (B1), we have used that \mathbf{u}_{cr} is directed along the y -axis and $\gamma_{cr1} = \gamma_{cr0}^3 u_{cr} v_{cr1y} / c^2$, where $\gamma_{cr0} = (1 - u_{cr}^2 / c^2)^{-1/2}$. From Equation (B1), we find the following equations for $v_{cr1x,y}$:

$$\gamma_{cr0} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) v_{cr1x} = -\frac{1}{m_{cr} n_{cr0}} \frac{\partial p_{cr1}}{\partial x} + F_{cr1x} + \omega_{ccr} v_{cr1y} \quad (\text{B3})$$

and

$$\gamma_{cr0}^3 \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) v_{cr1y} = -\frac{1}{m_{cr} n_{cr0}} \frac{\partial p_{cr1}}{\partial y} + F_{cr1y} - \omega_{ccr} v_{cr1x}, \quad (\text{B4})$$

where $\omega_{ccr} = q_{cr} B_0 / m_{cr} c$ is the cyclotron frequency of the cosmic-ray particles. Solutions of Equations (B3) and (B4) have the form

$$\Omega_{cr}^2 v_{cr1x} = \frac{1}{m_{cr} n_{cr0}} L_{1crx} p_{cr1} + \omega_{ccr} F_{cr1y} + \gamma_{cr0}^3 \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) F_{cr1x} \quad (\text{B5})$$

and

$$\Omega_{cr}^2 v_{cr1y} = \frac{1}{m_{cr} n_{cr0}} L_{1cry} p_{cr1} - \omega_{ccr} F_{cr1x} + \gamma_{cr0} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) F_{cr1y}, \quad (\text{B6})$$

where

$$\begin{aligned}\Omega_{cr}^2 &= \gamma_{cr0}^4 \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^2 + \omega_{ccr}^2, \\ L_{1crx} &= -\omega_{ccr} \frac{\partial}{\partial y} - \gamma_{cr0}^3 \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \frac{\partial}{\partial x}, \\ L_{1cry} &= \omega_{ccr} \frac{\partial}{\partial x} - \gamma_{cr0} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \frac{\partial}{\partial y}.\end{aligned}\tag{B7}$$

B.2. Equation for perturbed cosmic-ray pressure

From Equation (6) in the linear approximation, we obtain the perturbed cosmic-ray pressure

$$p_{cr1} = p_{cr0} \Gamma_{cr} \left(\frac{n_{cr1}}{n_{cr0}} - \frac{\gamma_{cr1}}{\gamma_{cr0}} \right).\tag{B8}$$

Using the linearized continuity equation (2) for cosmic rays and expression for γ_{cr1} , we find that p_{cr1} is given by

$$p_{cr1} = -p_{cr0} \Gamma_{cr} \left[\left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^{-1} \nabla \cdot \mathbf{v}_{cr1} + \gamma_{cr0}^2 \frac{u_{cr}}{c^2} v_{cr1y} \right].\tag{B9}$$

From Equations (B5) and (B6), we obtain the expression for $\nabla \cdot \mathbf{v}_{cr1}$ which is substituted together with the velocity v_{cr1y} into Equation (B9). As a result, we have

$$L_{2cr} p_{cr1} = -p_{cr0} \Gamma_{cr} \Phi_{cr1}.\tag{B10}$$

Here,

$$\begin{aligned}L_{2cr} &= \Omega_{cr}^2 - \gamma_{cr0} c_{scr}^2 L_{1cr} + \gamma_{cr0}^2 \frac{u_{cr}}{c^2} c_{scr}^2 L_{1cry}, \\ \Phi_{cr1} &= -L_{3crx} F_{cr1x} + L_{3cry} F_{cr1y},\end{aligned}\tag{B11}$$

where $c_{scr} = (p_{cr0}\Gamma_{cr}/m_{cr}n_{cr0})^{1/2}$ is the cosmic-ray sound speed defined by the rest mass and

$$\begin{aligned} L_{1cr} &= \gamma_{cr0}^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \\ L_{3crx} &= \omega_{ccr} \gamma_{cr0}^2 \left(\frac{u_{cr}}{c^2} \frac{\partial}{\partial t} + \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^{-1} - \gamma_{cr0}^3 \frac{\partial}{\partial x}, \\ L_{3cry} &= \omega_{ccr} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^{-1} \frac{\partial}{\partial x} + \gamma_{cr0}^3 \left(\frac{u_{cr}}{c^2} \frac{\partial}{\partial t} + \frac{\partial}{\partial y} \right). \end{aligned} \tag{B12}$$

B.3. Equations for cosmic ray velocities via \mathbf{F}_{cr1}

Substituting Equations (B10) and (B11) into Equations (B5) and (B6), we find

$$\Omega_{cr}^2 v_{cr1x} = \left[c_{scr}^2 \frac{L_{1crx}}{L_{2cr}} L_{3crx} + \gamma_{cr0}^3 \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \right] F_{cr1x} + \left(-c_{scr}^2 \frac{L_{1crx}}{L_{2cr}} L_{3cry} + \omega_{ccr} \right) F_{cr1y} \tag{B13}$$

and

$$\Omega_{cr}^2 v_{cr1y} = \left(c_{scr}^2 \frac{L_{1cry}}{L_{2cr}} L_{3crx} - \omega_{ccr} \right) F_{cr1x} + \left[-c_{scr}^2 \frac{L_{1cry}}{L_{2cr}} L_{3cry} + \gamma_{cr0} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \right] F_{cr1y}. \tag{B14}$$

B.4. Equations for cosmic-ray velocities via \mathbf{E}_1

From Equation (B2), we have

$$\begin{aligned} F_{cr1x} &= \frac{q_{cr}}{m_{cr}} \left[E_{1x} + u_{cr} \left(\frac{\partial}{\partial t} \right)^{-1} \left(\frac{\partial E_{1x}}{\partial y} - \frac{\partial E_{1y}}{\partial x} \right) \right], \\ F_{cr1y} &= \frac{q_{cr}}{m_{cr}} E_{1y}. \end{aligned} \tag{B15}$$

Substituting Equation (B15) into Equations (B13) and (B14), we obtain

$$\begin{aligned} v_{cr1x} = & \frac{q_{cr}}{m_{cr}\Omega_{cr}^2} \left[a_{crx} + \gamma_{cr0}^3 \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^2 \left(\frac{\partial}{\partial t} \right)^{-1} \right] E_{1x} \\ & + \frac{q_{cr}}{m_{cr}\Omega_{cr}^2} \left[-b_{crx} + \omega_{ccr} - \gamma_{cr0}^3 u_{cr} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} \right] E_{1y} \end{aligned} \quad (\text{B16})$$

and

$$\begin{aligned} v_{cr1y} = & \frac{q_{cr}}{m_{cr}\Omega_{cr}^2} \left[a_{cry} - \omega_{ccr} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} \right] E_{1x} \\ & + \frac{q_{cr}}{m_{cr}\Omega_{cr}^2} \left[-b_{cry} + \omega_{ccr} u_{cr} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} \right)^{-1} + \gamma_{cr0} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \right] E_{1y}, \end{aligned} \quad (\text{B17})$$

where

$$\begin{aligned} a_{crx} &= c_{scr}^2 \frac{L_{1crx}}{L_{2cr}} L_{3crx} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1}, \\ b_{crx} &= c_{scr}^2 \frac{L_{1crx}}{L_{2cr}} \left[L_{3cry} + L_{3crx} u_{cr} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} \right)^{-1} \right], \\ a_{cry} &= c_{scr}^2 \frac{L_{1cry}}{L_{2cr}} L_{3crx} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1}, \\ b_{cry} &= c_{scr}^2 \frac{L_{1cry}}{L_{2cr}} \left[L_{3cry} + L_{3crx} u_{cr} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} \right)^{-1} \right]. \end{aligned} \quad (\text{B18})$$

The operators $L_{1crx,y}$, L_{2cr} , and $L_{3crx,y}$ containing in Equation (B18) are given by Equations (B7), (B11), and (B12), respectively.

B.5. Perturbed cosmic ray current

We now find the components of the perturbed cosmic ray current $j_{cr1x} = q_{cr} n_{cr0} v_{cr1x}$ and $j_{cr1y} = q_{cr} n_{cr0} v_{cr1y} + q_{cr} n_{cr1} u_{cr}$. Using Equations (B16) and (B17) and the continuity equation (2) in the linear approximation, we find

$$4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{cr1x} = \varepsilon_{crxx} E_{1x} + \varepsilon_{crxy} E_{1y} \quad (\text{B19})$$

and

$$4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{cr1y} = \varepsilon_{cryx} E_{1x} + \varepsilon_{cryy} E_{1y}. \quad (\text{B20})$$

The components of the dielectric permeability tensor are the following:

$$\begin{aligned} \varepsilon_{crxx} &= \frac{\omega_{pcr}^2}{\Omega_{cr}^2} \left[a_{crx} + \gamma_{cr0}^3 \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^2 \left(\frac{\partial}{\partial t} \right)^{-1} \right] \left(\frac{\partial}{\partial t} \right)^{-1}, \\ \varepsilon_{crxy} &= \frac{\omega_{pcr}^2}{\Omega_{cr}^2} \left[-b_{crx} + \omega_{ccr} - \gamma_{cr0}^3 u_{cr} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} \right] \left(\frac{\partial}{\partial t} \right)^{-1}, \\ \varepsilon_{cryx} &= \frac{\omega_{pcr}^2}{\Omega_{cr}^2} \left[\left(a_{cry} \frac{\partial}{\partial t} - a_{crx} u_{cr} \frac{\partial}{\partial x} \right) \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^{-1} \right] \left(\frac{\partial}{\partial t} \right)^{-1} \\ &\quad - \frac{\omega_{pcr}^2}{\Omega_{cr}^2} \left[\omega_{ccr} + \gamma_{cr0}^3 u_{cr} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right) \left(\frac{\partial}{\partial t} \right)^{-1} \right] \left(\frac{\partial}{\partial t} \right)^{-1}, \\ \varepsilon_{cryy} &= \frac{\omega_{pcr}^2}{\Omega_{cr}^2} \left[\left(-b_{cry} \frac{\partial}{\partial t} + b_{crx} u_{cr} \frac{\partial}{\partial x} \right) \left(\frac{\partial}{\partial t} + u_{cr} \frac{\partial}{\partial y} \right)^{-1} \right] \left(\frac{\partial}{\partial t} \right)^{-1} \\ &\quad + \frac{\omega_{pcr}^2}{\Omega_{cr}^2} \gamma_{cr0} \left[1 + \gamma_{cr0}^2 u_{cr}^2 \frac{\partial^2}{\partial x^2} \left(\frac{\partial}{\partial t} \right)^{-2} \right]. \end{aligned} \quad (\text{B21})$$